ABSTRACT

ALTERNATE SUSPENSION SYSTEM FOR SPACE SHUTTLE AVIONICS SHELF

By

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August 2010

This thesis examines an equipment stowage shelf suspended from a frame in the cargo bay (mid fuselage) of the U.S. Space Shuttle, and three alternative designs. The first design is a conventional truss, representing the "tried and true" approach. The second is a cable dome type structure consisting of struts and pre-stressed cables. The third and fourth are double layer tensegrity systems consisting of contiguous struts of the order k=1 and k=2 respectively. The four options are compared to each other with an emphasis placed on weight, size, and approximate cost of each option

Results indicate the 4-Way Double Layer Tensegrity grid utilizing carbon fiber composite cables is the most efficient (lightest weight) tensegrity system, however for this particular application the most cost effective design was proven to be the optimized conventional truss. It was determined that the scale of the structure would have to increase substantially or tensegrity structures complexity must decrease for these alternative systems to compete with conventional designs.

ALTERNATE SUSPENSION SYSTEM FOR SPACE SHUTTLE AVIONICS SHELF

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TABLE OF CONTENTS

Page

AKNOWLEDGEMENTS iii

LIST OF TABLES vi

LIST OF FIGURES viii

CHAPTER

1. INTRODUCTION 1

    Comparison Variables 1
    Requirements 2

2. CONVENTIONAL STRUCTURAL DESIGN 8

    Design Philosophy 8
    History 9
    Configuration Overview 11
    Cost 14
    Analysis 15
    Results 17

3. TENSEGRITY 22

    Definition 22
    Origins 22
    Patents 26
    Noteworthy Structures 29
    Cable Dome 37
    Contiguous Grid 43
    Efficiency 46
    Technological Advancements 54
    Perceptions 56
    Knowledge Base 58

CHAPTER Page

4. CABLE DOMES 59

Configuration Overview 59
Analysis 63
Cost 65
Results... 66

5. DOUBLE LAYER TENSEGRITY GRIDS 69

Configurations Overview 69
Analysis 74
Bi-Directional Cost.. 77
Bi-Directional Results 79
4-Way Analysis 86
4-Way Cost 88
4-Way Results... 90

6. FINDINGS AND DISCUSSION.. 94

Description of Findings 94
Summary... 97

APPENDICES 99

A. PERSONAL CORRESPONDENCE.. 100

B. SPATIAL TRIANGULATION VS. RADIAL ORIENTED DOMES..... 131

C. MARGIN OF SAFETY CALCULATION FOR BASELINE DESIGN . 135

D. MARGIN OF SAFETY CALCULATION FOR CABLE DOME
DESIGN 138

E. MARGIN OF SAFETY CALCULATION FOR BI-DIRECTIONAL
DESIGN 141

F. MARGIN OF SAFETY CALCULATION FOR 4-WAY DESIGN........ 144

BIBLIOGRAPHY 148

LIST OF TABLES

TABLE Page

1. Liftoff (L/O) Transients (0-3seconds) 5
2. Liftoff (L/O) Vibro-Acoustic (3-6 seconds).. 5
3. Maximum Load Factors (0-6 seconds)... 5
4. Design Load Factors 5
5. Baseline Design... 13
6. Optimized Baseline Design 13
7. Baseline Engineering Design Hours.. 15
8. Strut Element Forces, Baseline Structure... 17
9. Margin of Safety (M.S.) Summary.. 21
10. Cable Material Properties 56
11. Cable Dome Preliminary Sizing... 64
12. Cable Dome Engineering Design Hours 66
13. Element Forces, Cable Dome Structure 67
14. Margin of Safety (M.S.) Summary... 67
15. Cable Dome Final Sizing... 68
16. Bi-directional Grid Preliminary Sizing 76
17. Bi-directional Engineering Design Hours 78
18. Element Forces, Bi-Directional Structure 79

TABLE Page

19. Bi-Directional Grid Final Sizing 80

20. Margin of Safety (M.S.) Summary 81

21. 4-Way Grid Preliminary Sizing 89

22. 4-Way Engineering Design Hours 90

23. Element Forces, 4-Way Structure 91

24. Margin of Safety (M.S.) Summary 91

25. 4-Way Grid Final Sizing 92

26. Design Weight Summary 95

27. Design Cost Summary 95

28. Shelf Displacement Summary 96

29. Margin of Safety (M.S.) Calculations, Baseline Design (Optimized) 136

30. Margin of Safety (M.S.) Calculations, Cable Dome............................ 139

31. Margin of Safety (M.S.) Calculations, Bi-Directional......................... 142

32. Margin of Safety (M.S.) Calculations, 4-Way Grid............................. 145

33. Margin of Safety (M.S.) Calculations, 4-Way Grid............................. 146

LIST OF FIGURES

FIGURE Page

1. Shelf orientation 3
2. Shelf side view 7
3. Isometric view of baseline structure 11
4. Top view of baseline structure 12
5. Node point and element member, baseline structure 16
6. X-Piece 24
7. Kenneth Snelson’s U.S. Patent 3,169,611: Continuous Tension, Discontinuous Compression Structures 27
8. Experimental Planar Structures from 1961: Woven Planes 29
9. Experimental Planar Structures from 1961: Woven Planes on rooftop 30
10. Needle Tower, 1968, 60 x 20 x 20 feet, Collection: Hirshhorn Museum and Sculpture Garden, Washington, D.C. 31
11. Needle Tower, 1968 32
12. Zig-Zag Tower, 1997 painted stainless steel, 45.5” x 9” x 7.75” 33
13. Messeturm in Rostock (Tower in Rostock) 34
14. Georgia Dome cost vs. post length 39
15. Georgia Dome optimal configuration 39
16. Efficiency ratio vs. span of double layer space trusses, adjusted for imposed load of 100 kg/m^2 49

FIGURE Page

17. Efficiency ratio vs. span for braced domes 49

18. Structural efficiency of constructed cable roofs and domes 50

19. Structural efficiency of designed bar-tendon double-layer grids adjusted for imposed load of 100 kg/m^2. Span = 27-30 m. 50

20. Bar-tendon grids with continuous bar chords proposed by Wang 51

21. Stabilized form of the CP grids: (a) CP-a grid; (b) CP-b grid 52

22. Special CP configurations by connecting edges: (a) a CP truss; (b) CP grid; (c) two-way configuration of (a)............ 53

23. Flowchart illustrating general approach to tensile membrane structure design and engineering..................................... 55

24. Cable dome tensegrity structure (k=1) isometric view........................ 60

25. Cable dome tensegrity structure top view 61

26. Cable dome tensegrity structure side view. 61

27. Cable dome tensegrity structure view A 62

28. Bi-directional double layer tensegrity grid (k>1) isometric view 70

29. Bi-directional double layer tensegrity grid (k>1) top view 71

30. Bi-directional double layer tensegrity grid from top to bottom: the upper layer of cables, the bracing of cables, the woven struts, the lower layer of cables 72

31. Bi-directional double layer tensegrity grid (k>1) cross sectional view. 73

32. Bi-directional double layer tensegrity grid (k>1): (a) simplex; (b) detail view.. 74

33. 4-Way double layer tensegrity grid isometric view. 82

FIGURE Page

34. 4-Way double layer tensegrity grid (k>1) top 83

35. 4-Way double layer tensegrity grid (k>1) simplex 84

36. 4-Way double layer tensegrity grid side view...................................... 85

37. 4-Way double layer tensegrity grid (k>1):
(a) simplex; (b) detail view.. 86

38. Baseline maximum displacements 137

39. Cable dome maximum displacements.. 140

40. Bi-directional grid maximum displacements 143

41. 4-Way grid maximum displacements.. 147

CHAPTER 1

INTRODUCTION

Engineers working in the Aerospace field under deadlines and strict budgets often miss the opportunity to design something that is considered new or “innovative,” favoring instead to use the “tried-and-true” design over those that may, in fact, be more efficient. This thesis examines an electronic equipment stowage shelf suspended from a frame in the cargo bay (mid fuselage) of the United States Space Transportation System (STS), the Space Shuttle, and three alternative designs.

Four different designs are examined and evaluated. The first design is a conventional truss, representing the “tried and true” approach. The second is a cable dome type structure consisting of struts and pre-stressed wiring. The third and fourth are double layer tensegrity systems consisting of contiguous struts of the order k=1 and k=2 respectively.

Comparison Variables

The four options are then compared to each other. As this is a space launch vehicle, emphasis is placed on the weight, size and approximate cost of each option. Points are awarded based on percentage above/below the existing design and are later tabulated to determine which option is more efficient.

Cost

The cost of each a typical design consists of a sum of the following: material, manufacturing, assembly and tooling cost (if applicable) plus engineering design hours and any preliminary testing (if required). An hourly engineering design rate of $250 an hour shall be used. Each of the alternate designs will be compared to the baseline. Engineering design hours and material costs are approximated.

Weight

The cost of launching the U.S. Space Shuttle is approximately $450 million per mission, or approximately $19,000 per pound [1]. Modifications made to the Space Shuttle may have a direct impact on the cargo carrying capability, depending on the location with respect to the vehicle's center of gravity. For this reason it is essential for the structure to weigh a minimum. The weight of the conventional truss support structure flying on the vehicle is 2.44 lbs. (see Table 4a). The weight of this structure optimized is 1.082 lbs. (see Table 4b). Alternate designs will be compared to this baseline design weight.

Size

Due to size and space limitations in the cargo bay, the support structure was designed not to encroach beyond the defined installation envelope. See envelope constraints defined under requirements.

Requirements

The loads and environments that a typical shelf and vehicle are subjected to, as well as typical weight, are shown below. The flight coordinate system used is shown in Figure 1. The volume that the shelf is to be installed is limited to the space between frames on the theoretical vehicle stations Xo 1300.00, and Xo = 1356.00. Static and Dynamic analysis are performed on all options, verifying each system meets the requirements stated below.

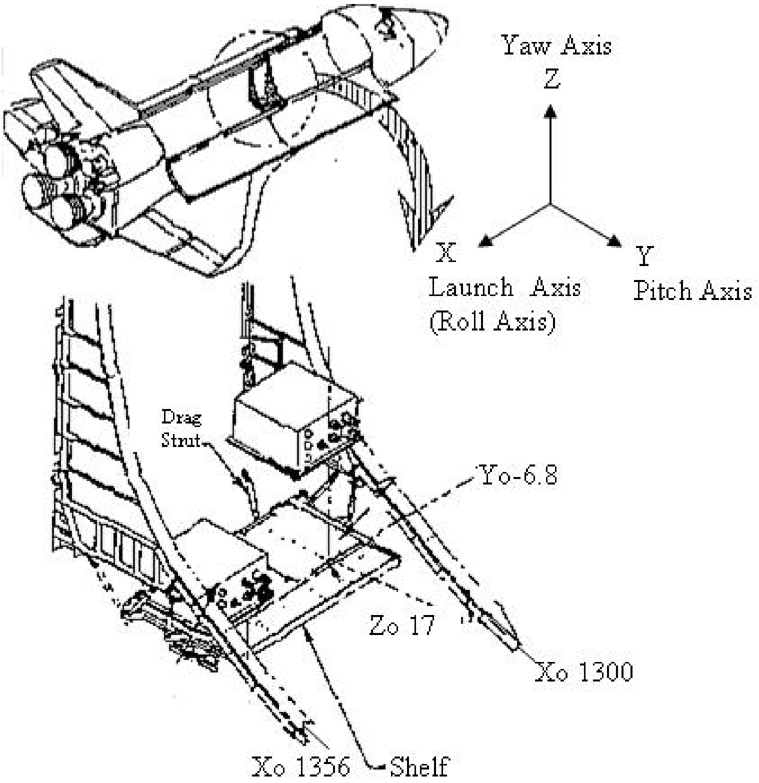


FIGURE 1. Shelf orientation (Copyright © [2010] Boeing. All rights reserved, Reproduced with permission).

Flight Loads

Loads and stresses were derived from typical liftoff (L/O) condition (transient plus random vibration, see Table 1) for T (time) equal to zero to plus three seconds; L/O condition (random vibration, see Table 2) for T equal to plus three to plus six seconds; landing condition and crash condition. The crash loads are 9g ultimate applied to a twenty degree cone in the forward (-Xo) direction. The minimum margin of safety (M.S) is 0.00.

A conservative combined L/O and vibro-acoustic load factor is achieved by calculating the root sum square (RSS) of 100% of the transient load factor with 50% of the vibro-acoustic load factor. This operation is applied for each axis and only the maximum values are shown. An Xo axis steady state acceleration of -1.5 is directly added to the Xo axis dynamic load factors.

Nx = RSS of 100% (6.10-1.5) and 50% (2.5) = +3.27 / -6.27

Ny = RSS of 100% (2.70) and 50% (4) = 3.36

Nz = RSS of 100% (4.67) and 50% (5) = 5.30

To determine what the controlling load factor is we must add the Xo steady state acceleration to the Table 2 Xo value as shown below. From this we can determine the greatest load factors (purely random or transient + random). We find that Nx and Nz maximums are determined by a combination of transient and random load factors while Ny is purely random.

Nx = +/-2.5 -1.50 = +1.00 / -4.00

Ny = +/-4

Nz = +/-5

Load in one axis may contribute to load in another, orthogonal, axis. For this reason a conservative loading could be realized by combining 100% of the primary load axis with no more than 30% of the loads from each of the other two axis which represents the equivalent static load factor. Only maximum load factor calculations are shown below and summarized in Table 3.

Nx = RSS of 100% (3.27) and 30% (3.36) and 30% (5.30) = 3.77

-Nx = RSS of 100% (6.27) and 30% (3.36) and 30% (5.30) = -6.55

Ny = RSS of 30% (1.00) and 100% (4.00) and 30% (5.00) = 4.28

Ny = RSS of 30% (-4.00) and 100% (4.00) and 30% (5.00) = -4.44

Nz = RSS of 30% (3.27) and 30% (3.36) and 100% (5.30) = 5.48

Nz = RSS of 30% (-6.27) and 30% (3.36) and 100% (5.30) = -5.72

TABLE 1. Liftoff (L/O) Transients (0 - 3 seconds)

+Nx	*-Nx*	*+/-Ny*	*+/-Nz*
6.10	-6.10	2.70	4.67

TABLE 2. Liftoff (L/O) Vibro-Acoustic (3-6 seconds)

+Nx	*-Nx*	*+/-Ny*	*+/-Nz*
2.5	-2.5	4	5

Note: A steady state of Nx = -1.5g is added for liftoff dynamics.

TABLE 3. Maximum Load Factors (0-6 seconds)

+Nx	*-Nx*	*+Ny*	*-Ny*	*+Nz*	*-Nz*
3.77	-6.55	4.28	-4.44	5.48	-5.72

TABLE 4. Design Load Factors

+Nx	-Nx	+/-Ny	+/-Nz
3.8	-6.8	4.8	6

The typical design loads for liftoff are shown in Table 4. For enveloping purposes the design load factors were increased slightly in the Xo, Yo and Zo axis resulting in an inclusion of landing load factors under the liftoff condition. This simplification in the analysis load case allows for a clear comparison between the baseline and alternate designs.

Equipment Stowage Shelf Properties

The equipment stowage shelf, shown in Figure 1 and Figure 2, is approximately 50 inches long by 22 inches wide by 2 inches tall and is located between Xo 1302.64 and Xo 1350.80, Yo -6.80 and Yo -43.80, and Zo 17.00 to Zo 19.00. The natural frequency requirement for a typical avionics shelf is between 26 and 30 Hz and the shock Spectra between 20 and 30 hz. The approximate weight of the shelf is 15 lb., and avionics boxes weigh approximately 263 lb. total. The center of gravity is located at Xo 1328.00 , Yo -16.00 and Zo 18.

Envelope Constraints

The installation envelope (see Figure 1 and Figure 2) lies between stations Xo 1300.00 and Xo 1356.00, between buttock lines Yo -88.80 and Yo -6.80, and between water lines Zo 10.00 and Zo 30.00.

The design must also utilize existing primary structure to mount the support system. Primary structure consists of theoretical frames Xo 1356.00 and Xo 1300.00, and theoretical sidewall at Yo-88.80.

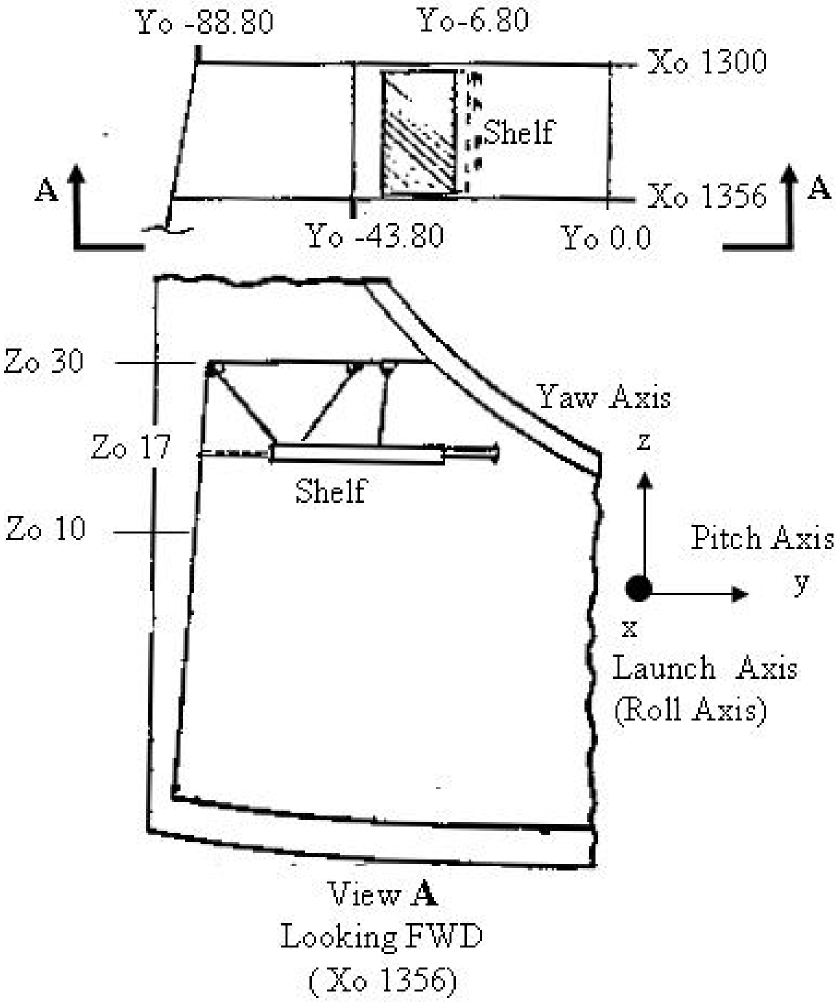


FIGURE 2. Shelf side view (Copyright © [2010] Boeing. All rights reserved, Reproduced with permission).

CHAPTER 2

CONVENTIONAL STRUCTURAL DESIGN

Design Philosophy

Engineers working in the Aerospace field are under deadlines and strict budgets that affect their final product. For the United States space program the mitigating factors are schedule, cost, and weight. The order of priority for these may vary depending on the criticality of the part (the safety factor required to be used) and the required schedule (installation date and where/how it gets installed). All of the above must be determined with an overwhelming emphasis on safety for the vehicle and, more importantly, the crew.

Lessons Learned

Most designs are not unique, and for this reason most engineers will use historical designs on either their current or past programs as a template for the new design. This may yield a schedule and cost savings by utilizing the lessons learned from the previous design.

Requirements

Typical avionics shelf requirements are defined in the introduction of the thesis. Avionics boxes installed on the shelf have certain operating limitations. The boxes will dictate how stiff the support structure must be to allow the box to not only survive, but to allow all of the inner internal avionics to work under the extreme launch, landing, possible crash landing, and on-orbit environments experienced by the Space Shuttle.

The design is also constrained by the available area to mount support structure. The area defined in the requirements section (see Introduction) has additional structure which also must be avoided (four primary support struts are not shown). For each design shown great care has been taken to avoid interference, and, as a result, optimization of the design has suffered to a small degree. This, however, presents a common design problem and ensures that the end product will represent a realistic view of those designed reviewed.

Design Optimization

Multidisciplinary design optimization (MDO) may be used to arrive at a minimum weight for a given envelope, rigidity and material requirement. Finite element analysis (FEA) programs such as HyperSizer (Collier Research Corporation) and MSC Insight (MSC Software Corporation) allow users to input these variables and the computer program provides an optimum design. The potential problem with this, as is the case in all finite element modeling (FEM), is user error in the definition of boundary conditions and a reliance on the program instead of it performing as a useful tool.

History

The avionics shelf that is attached below the cargo bay in the mid fuselage of the Space Shuttle must be suspended. Traditional design for suspension support structure includes the space frame. A space frame is universally accepted as one of the most efficient support structures. For this reason, it is commonly used in almost every application imaginable and as a result is a lower cost option for most engineering designs.

Space Frames are reviewed in detail in G.S. Ramaswamy et al.'s book *Steel Space Frames*. Ramaswamy prefers to use the American Society of Civil Engineers 1976 Task Committee on Latticed Structures definition of a space frame:

> . . . a structural system in the form of a network of elements (as opposed to a continuous surface) . . . another characteristic of lattice structural systems is that their load-carrying mechanism is three dimensional in nature. [2]

Ramaswamy notes that the key advantages of space frames are their light weight and ability to distribute load. Due to the nature of the geometry of the space frame loads are distributed to other parts of the frame. This directly results in a decreased stress for each frame member, and therefore allows for a more efficient, light weight, structure. The structural stiffness is generally higher, resulting in minimal deflections. And finally the assembly and installation of space frames is, because of their simplicity, accomplished quickly with very little complexity [2].

A less traditional design includes the use of suspended cables. Suspended cables are utilized in automobile and pedestrian bridge design. Historical structures that utilize such cables include the Brooklyn Bridge and the Golden Gate Bridge. Cable stay bridge design has been developed and has "become a widely used type of long-span bridges, due to the superior self-balancing structural system, higher overall stiffness and better aerodynamic behaviour in comparison to suspension bridges" [3]. Other examples of cable-stayed bridges include the Sutong Bridge (1088 m) in China, the Stonecutters Bridge (1018 m) in Hong Kong, China and the Tatara Bridge (890 m) in Japan [3].

Configuration Overview

The baseline design of the shelf suspension system consists of six struts attached to the main mid fuselage frames (located at Xo 1300 and Xo 1356) and one drag strut attached to the sidewall (located at Yo -88.80). See Figure 1 and Figure 2 for an overview of the installation and reference dimensions for each element. The MSC PATRAN model isometric view is shown in Figure 3. A top view of the model is also shown in Figure 4. Baseline strut dimensions are shown in Table 5. Overall length and strut diameter shown are for a uniform cross section (the design was simplified for a more direct comparison with the alternate designs).

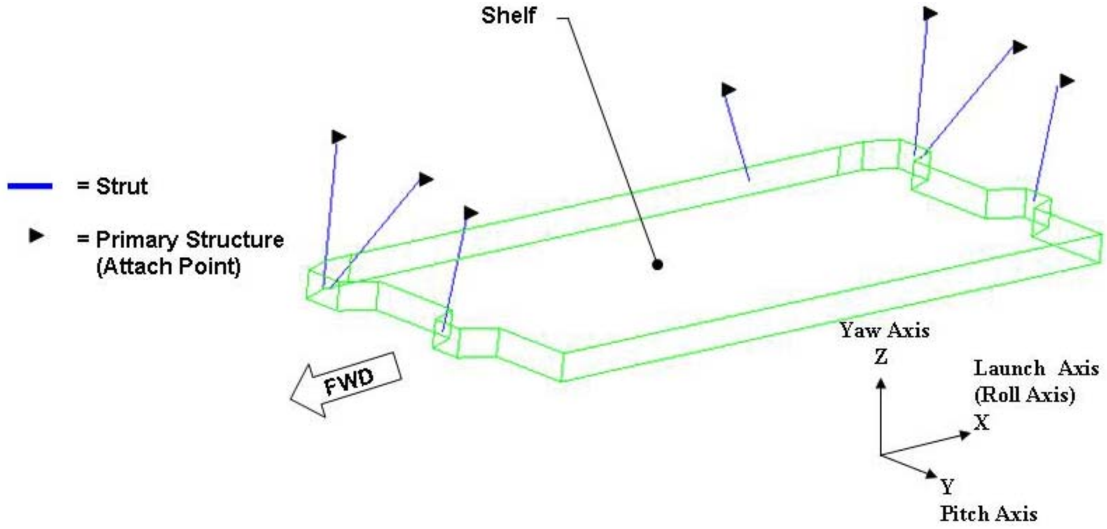


FIGURE 3. Isometric view of baseline structure (Biele, F.).

The material selected for the struts was 6Al-4V Titanium. Material allowable properties for the struts were B-Basis based on the definition from the *Metallic Material*

Properties Development and Standardization Handbook (MMPDS) that states "least 90 percent of the population of values is expected to equal or exceed the B-basis mechanical property allowable, with a confidence of 95 percent" [4]. In addition the shelf supports are not primary structure and contain redundant load paths. Federal Aviation Regulations (FARs) Part 25, *Airworthiness Standards: Transport Category Airplanes*, section 613 states "for redundant structure in which the failure of individual elements would result in applied loads being safely distributed to the load carrying members, 90 percent probability with 95 percent confidence" [5]. The total weight of the baseline design is 2.44 lbs. and the optimized baseline support system analyzed is 1.082 lbs. (see Table 5 and Table 6).

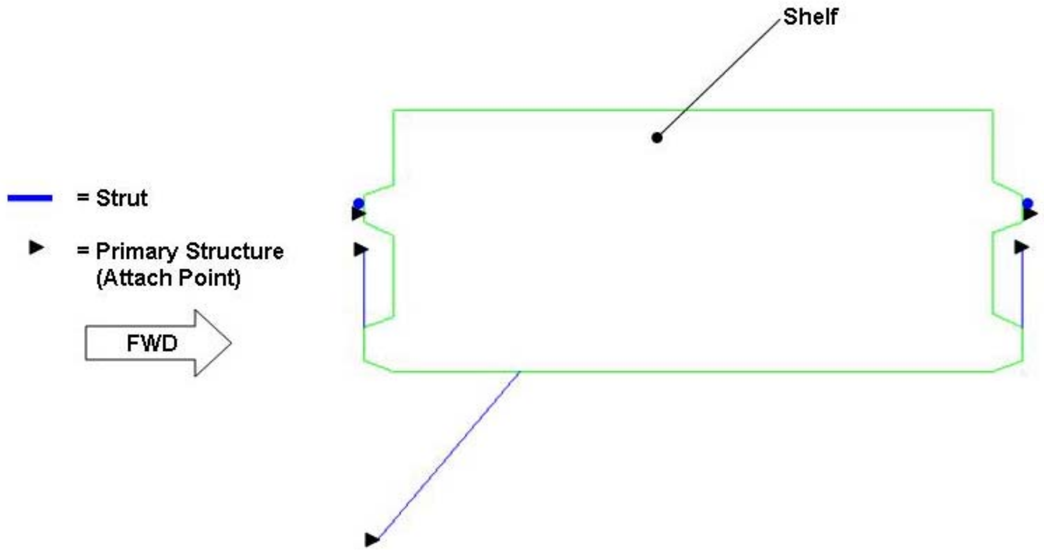


FIGURE 4. Top view of baseline structure (Biele, F.).

TABLE 5. Baseline Design

Description	Strut Dash Number	QTY	Element No.	Diameter (inches)	Pin to Pin Length (inches)	Wall Thickness (inches)	Sectional Area (inches^2)	Height of Uniform Diameter (inch)	I	EI/A	EA	Ti Volume (inches^3)	Individual Weight 6AL-4V (lbs)	Total Weight (lbs)
Side Struts	1356 Inbd	1	2072	1.00	8.5	0.051	1.52E-01	3.38	1.72E-02	1862854	2508824	1.292	0.2068	0.2068
Side Struts	1300 Inbd	1	2071	1.00	8.6	0.051	1.52E-01	3.39	1.72E-02	1862854	2508824	1.308	0.2092	0.2092
Side Struts	Center	2	2070/4	1.00	12.4	0.051	1.52E-01	7.00	1.72E-02	1862854	2508824	1.885	0.3017	0.6033
Side Struts	Outbd	2	2069/75	1.00	14.4	0.051	1.52E-01	9.00	1.72E-02	1862854	2508824	2.190	0.3503	0.7006
Drag Strut	Drag	1	2068	1.25	18.5	0.065	2.42E-01	10.50	4.26E-02	2904928	3992689	4.477	0.7163	0.7163
					TOTAL		1.15E+00					15.227	1.7843	2.4363

TABLE 6. Optimized Baseline Design

Description	Strut Dash Number	QTY	Element No.	Diameter (inches)	Pin to Pin Length (inches)	Wall Thickness (inches)	Sectional Area (inches^2)	Height of Uniform Diameter (inch)	I	EI/A	EA	Ti Volume (inches^3)	Individual Weight 6AL-4V (lbs)	Total Weight (lbs)
Side Struts	1356 Inbd	1	2072	1.00	8.5	0.021	6.46E-02	3.38	7.74E-03	1977694	1065702	0.5490	0.0878	0.0878
Side Struts	1300 Inbd	1	2071	1.00	8.6	0.021	6.46E-02	3.39	7.74E-03	1977694	1065702	0.5555	0.0889	0.0889
Side Struts	Center	2	2070/4	1.00	12.4	0.021	6.46E-02	7.00	7.74E-03	1977694	1065702	0.8009	0.1281	0.2563
Side Struts	Outbd	2	2069/75	1.00	14.4	0.021	6.46E-02	9.00	7.74E-03	1977694	1065702	0.9301	0.1488	0.2976
Drag Strut	Drag	1	2068	1.25	18.5	0.031	1.19E-01	10.50	2.21E-02	3066777	1958841	2.1963	0.3514	0.3514
					TOTAL		5.06E-01					6.7626	0.8051	1.0820

Cost

The baseline design hours are approximated by first determining the hardware count. The total number of struts is shown in Table 5. The struts are installed by an installation drawing that contains the struts and any attach hardware to primary structure that is required. The hours required to design each detailed piece of hardware are approximated as one hundred hours per detailed drawing, twenty hours per page per installation drawing, and layout drawings are estimated to require 200 hours.

Additional hours include planning and scheduling, as well as design engineering support hours for manufacturing. Each article of released engineering (detailed and installation drawings) require half of an hour per document to maintain and track and two hours per week to update and track the total list. A total of 5% of all hardware manufactured is expected to not conform to drawing requirements and will require four hours to disposition. In addition 3% of the parts will require drawing clarification and will require four hours to disposition.

An itemized list of design hours for the baseline are shown in Table 7. The total design hours are 1,366 which will require approximately four months to complete (two persons working full time). Each of the alternate designs will be compared to this total. The hours may be converted into a total cost by multiplying by $250 (United States) an hour.

Material cost was approximated to be thirty dollars per inch cubed for titanium tubing. The total volume of titanium used in the baseline design is shown in Table 5.

Utilizing the total titanium volume we can approximate the total material cost as $203. The total cost of both engineering design and material is approximately $342,000.

TABLE 7. Baseline Engineering Design Hours

Product	Quantity	Hours
Layout	1 Drawing	200
Strut	5 Drawings	500
Attach Brackets	4 Drawings (3 Common Brackets)	400
Installation	1 Drawing (11 pages)	220
Maintain and Track	11 Drawings	6
Update Schedule	16 Weeks	32
Non-Conformance	1 Part (5% of 12 parts)	4
Drawing Clarification	1 Part (3% of 12 parts)	4
Total		1366

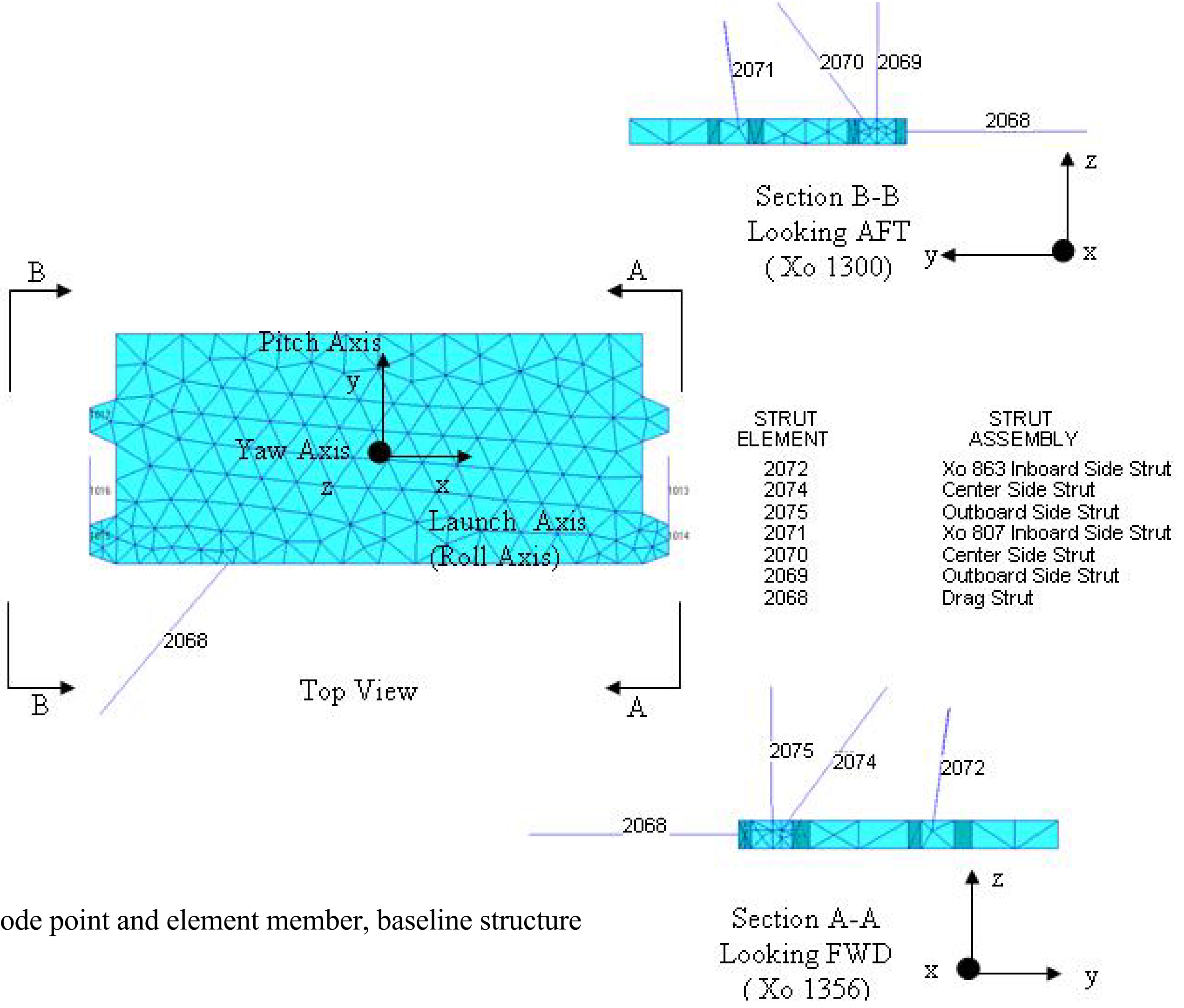
Note: Design hours are rounded up to the nearest whole hour. Total part quantities are rounded to the nearest whole part for tracking and disposition purposes.

Analysis

The baseline design was modeled in MSC PATRAN 2008 and analyzed using MSC NASTRAN (MD version R3b). The NASTRAN finite element model reference data is shown in Figure 5. Struts were modeled as Patran PROD elements and the shelf is a tet10 solid with a load applied at the center of gravity through an Patran MPC (RBE2).

Results

Loads for each strut, obtained from NASTRAN, are shown in Table 8. NASTRAN load data was used to verify the margin of safety (MS) for tension, compression and bending (local and Euler) and is shown in Table 9. Hand calculations for the drag strut (element 2068) are shown below and are typical for all struts analyzed. Also reference Appendix C for a table of calculations for all baseline design struts.



STRUT ELEMENT	STRUT ASSEMBLY
2072	Xo 863 Inboard Side Strut
2074	Center Side Strut
2075	Outboard Side Strut
2071	Xo 807 Inboard Side Strut
2070	Center Side Strut
2069	Outboard Side Strut
2068	Drag Strut

FIGURE 5. Node point and element member, baseline structure (Biele, F.) .

TABLE 8. Strut Element Forces, Baseline Structure

	Axial Load (lbs.)		
Strut No.	Liftoff (Limit Load)	Crash Landing (Ultimate Load)	Maximum
2072	1354	603	1896
(Xo1356 INBD)	-1226	-223	-1716
2074	1910	932	2674
(Xo1356 Center)	-2269	-2152	-3177
2075	1812	1718	2537
(Xo1356 OUTBD)	-1483	-724	-2077
2069	2272	2155	3181
(Xo 1300 OUTBD)	-1593	-724	-2230
2070	2101	935	2941
(Xo1300 Center)	-2996	-2841	-4194
2071	1086	483	1520
(Xo1300 INBD)	-983	-179	-1376
2068	2870	0	4018
(Drag)	-4093	-3882	-5731

Loads.

$P_t = 2870.24 \times 1.4$ (factor of safety) = 4018 lbs. ultimate Liftoff

$P_c = -4093.35 \times 1.4$ (factor of safety) = -5731 lbs. ultimate Liftoff

Section properties.

$D_1 = OD = 1.250$ in

t = tube wall thickness: 0.031 in.

$D_2 = ID = 1.188$ in.

$R = 0.610$ in.

$A = \pi \; (D_1^2 - D_2^2) / 4 = 0.119 \text{ in}^2$

$I = \pi \; (D_1^4 - D_2^4) / 64 = 0.022 \text{ in}^4$

$\rho = \sqrt{(I / A)} = \sqrt{(0.043 / 0.242)} = 0.431$

$L / \rho = 18.50 / 0.422 = 42.9$

Local buckling.

$Fcr = C\,E\,(t / r)$ (Bruhn, eq. c8.5 [6])

$C = 1 / \sqrt{(3\,(1 - \nu^2)\,)}$

ν = Poisson’s ratio

$Z = (L^2 / (r\,t)\,)\,\sqrt{(1 - \nu^2)}$ (Bruhn, eq. c8.5 [6])

$Z = 16600$

$Kc = (4\,\sqrt{3} / \pi^2)\,Z$ (Bruhn, eq. c8.3 [6])

Using Bruhn [6] Figure C8.7 and $Z = 16600$; $Kc = 3200$

$\sigma_{cr}/\eta = Kc\,(\pi^2\,E) / (\,12(3\,(1 - \nu^2)\,)\ \ (t / L)^2$ (Bruhn, eq. C8.2 [6])

$\sigma_{cr}/\eta = 138087$ psi Therefore material failure is a conservative approximation.

$P_E = \pi^2\,E\,I / L^2$ (Niu, eq. 10.2.1 [7])

$P_E = \pi^2\,E\,I / L'^2$

Where L' = Effective Length = $L / \sqrt{C}$

C = column end fixity = 1; For uniform, axially loaded beam with pinned ends [7].

$F_{CR} = \pi^2\,E\ / (L' / \rho\,)^2$

$P_E = F_{CR} = 88438$ and may be used for $\sigma_{cc} = F_{CR}$

Column allowable.

$Cc = \sqrt{(2\,\pi^2\,E\ / \sigma_{cc}\,)} = 60.69$

$L/ \rho = 42.91 < 60.69$ => Short Column

Johnson-Euler column formula (Bruhn, eq. 10.8.1 [6])

$Fc = \sigma_{cc}\,[\,1 - (\,\sigma_{cc}\,(L' / / \rho\,)^2\ /\ 4\,\pi^2\,E\,)\,] = 66328$ psi

$F_{CR} = A\ f_{CR} = 0.1187\ in^2 \times 66328\ psi = 7873$ lbs. ultimate

Beam–Column.

$M = P\,(e + \delta)$ (Timoshenko, [8])

e = eccentricity

δ = deflection measure from the axis of the column

A beam column under axial compression with equal end moments produces a maximum stress level at the tube mid point. The maximum eccentricity is assumed to occur at the tube ends and the classical assumption of e = (0.001) (tube length) is used (e = 0.030 minimum). In addition 0.026 inches is added to the manufacturing eccentricity to account for installation tolerances.

$Mend = Pc\,(0.001L + 0.026) = 255.0$ in.lb.

The maximum moment at x = L / 2 is calculated using the Approximation Method

$M_{MAX} = Mo / (1 - (P/P_{CR}))$ (Niu, equation 10.6.5 [7])

or

$M_{MAX} = M / \cos(L / 2J) = 639$ (Bruhn, Table A5.1 [6])

Where $J = \sqrt{(E\,I / P)} = 7.97$

The maximum bending stress at x = L / 2

$\sigma_b = M_{MAX}\,c / I = 17660$ psi

Allowable bending stress. Bending stresses are calculated by utilizing Cozzone simplified procedure:

$M_b\,c / I = fm + fo(2\,Q / (I/c) - 1)$ (Bruhn, eq. c3.3(1) [6])

$K = 2\,Q / (I / c)$ (Bruhn, eq. c3.3(2) [6])

$Fb = fm + fo (k – 1)$ (Bruhn, eq. c3.3(3) [6])

$Q_{\text{tube max}} = 2/3 (R_{\text{outer}}^3 – R_{\text{inner}}^3) = 0.0230$

$K = 2 Q c / I = 1.273$ (confirmed with check of Bruhn Figure C3.7 [6])

fo is found by plotting Fty on Figure c3.20 (Bruhn), strain, ϵ, is 0.01 in. / in. and

fo = 40 ksi. . (Bruhn, Figure c3.20 [6])

$F_{\text{byield}} = 120$ ksi + 40 ksi (1.273 – 1) (Bruhn, eq. C3.3(3) [6])

$F_{\text{byield}} = 131$ ksi

Then $M_{Yb} = F_{\text{byield}}$ x I / c = 131 ksi x 0.022 in^2/ 0.609 = 4728 in.lb.

Substituting Fty = fm in equation c3.3(3) fo is found using figure c3.20 [6]

Fo = 120 ksi

Fb (Ult) = 130 ksi + 112 ksi (1.273 -1)

Fb (Ult) = 160.5 ksi

M_{Ult} = Fb I / c = 160.5 ksi (0.022 in^4) / 0.609 = 5798 in.lb.

Margin of safety (M.S.).

Rc + Rb = 1 (Bruhn, eq. c4.11 [6])

Rbending = Rb = σ_b / Fb(Ult) = 17660 ksi / 160.5 ksi = 0.1100

Rcomp = Rc = Pc / Pcr = 5731 psi / 7873 psi = 0.7278

M.S. = 1 / (Rb + Rc) - 1 (Bruhn, eq. c4.14 [6])

M.S. = 1 / (.1100 + 0.7278) – 1 = 0.19 (for compression and bending)

Compression and tension stress check.

$\sigma_c = (Pc / A) + (M_{MAX}\, c / I) = (Pc / A) + \sigma_b$ = (5731 lbs / .119 in^2) + 17660 psi

σ_c = 65819 psi ultimate

M.S. = $(F_{cy} / \sigma_c) - 1 = (137 \text{ ksi} / 65.819 \text{ ksi}) - 1 = 1.08$ (compression)

$\sigma_T = (P_T / A) + (M\,c / I) = (4018 \text{ lbs} / .119 \text{ in}^2) + 255.0 \text{ in-lbs } (.610 \text{ in}) / 0.022 \text{ in}^4$

$\sigma_T = 40835$ psi ultimate

M.S. = $(F_{TU} / \sigma_T) - 1 = (133 \text{ ksi} / 40.835 \text{ ksi}) - 1 = 2.25$ (compression)

TABLE 9. Margin of Safety (M.S.) Summary

ELEMENT No.	Description	Failure Mode	M.S.
2072	Xo 1356 Inboard Side Strut	Tension	3.02
2074	Center Side Strut	Comp & Bending	0.21
2075	Outboard Side Strut	Comp & Bending	0.82
2071	Xo 1300 Inboard Side Strut	Tension	4.01
2070	Center Side Strut	Comp & Bending	0.21
2069	Outboard Side Strut	Comp & Bending	0.82
2068	Drag Strut	Comp & Bending	0.19

CHAPTER 2

TENSEGRITY

Definition

Tensegrity can be interpreted as an attempt to manipulate the conventional rigid truss structure in a way that causes an efficient distribution of the load and, as a result, a reduction in the weight of the overall structure. While it is true that "rigid" truss structure may be customized to give a similar result, the ability of the structure to deform without a yielding of the structure is certainly limited with respect to the tensegrity system, whether it be a triangulated contiguous (strut contacting strut) system or the more traditional tensegrity grids (k=1, more on this later).

The most agreed upon and concise definition of tensegrity is arguably written by Anthony Pugh. This definition can be interpreted as a merging of the ideas of David Emmerich, Buckminster Fuller and Kenneth Snelson [9]. It is not surprising that Pugh's definition is accepted because of the question of who, among Emmerich, Fuller and Snelson, invented tensegrity. Pugh writes that "a tensegrity system is established when a set of discontinuous compression components interacts with a set of continuous tensile components to define a stable volume in space" [10].

Origins

The credit for the invention of tensegrity could be compared to the somewhat more "explosive" physicist Lise Meitner's subjugation to Hahn Otto. Otto, who even after

WWII refused to credit Meitner, was the 1944 recipient of the Nobel Prize in Chemistry for the discovery of fission. As Elisabeth Crawford et al. note in their text, Meitner was an integral part of the team, and together with her nephew provided the most important interpretation of the experimental data, which led to the final discovery of fission" [11].

Parallels may be found between the history of the discovery of fission and the discovery of tensegrity. Snelson, while a student of Fuller at Black Mountain College, designed and built an amazing suspended "X" structure (see X-piece in Figure 6). As Fuller notes in a letter to Snelson dated December 22, 1949:

> In all my public lectures I tell of your original demonstration of discontinuous—pressure-(com-pressure) and continuous tension structural advantage; -in which right makes light in a prototype structure, the ready reproduction of which, properly incorporated in fundamental structures, may advance the spontaneous good will and understanding of mankind by many centuries. The event was one of those 'It happened' events, but demonstrates how the important events happen where the atmosphere is most favorable. If you had demonstrated this structure to an art audience it would not have rung the bell that it rang in me, who had been seeking this structure in Energetic Geometry. That you were excited by the latter, E.G., into spontaneous articulation of the solution, also demonstrates the importance of good faith of colleagues of this frontier. The name of Ken Snelson [his underline] will come to be known as a true pioneer of the realized good life and good will. [12]

Unfortunately Fuller never publicly acknowledged Snelson's contribution, except for a 1959 Museum of Modern Art showing of Fuller's "mast" structure.

Meitner and Snelson found themselves outside the scientific community for various reasons. As Crawford et al. note "Meitner's exclusion from the chemistry award [Nobel Prize] may well be summarized as a mixture of disciplinary bias, political obtuseness, ignorance and haste" [11]. Snelson's exclusion could be described as part pride and part glory, by Fuller, and part professional bias. Snelson explains that artists use their work, in his case sculpture, as scientists or engineers use publications, with the exception of his

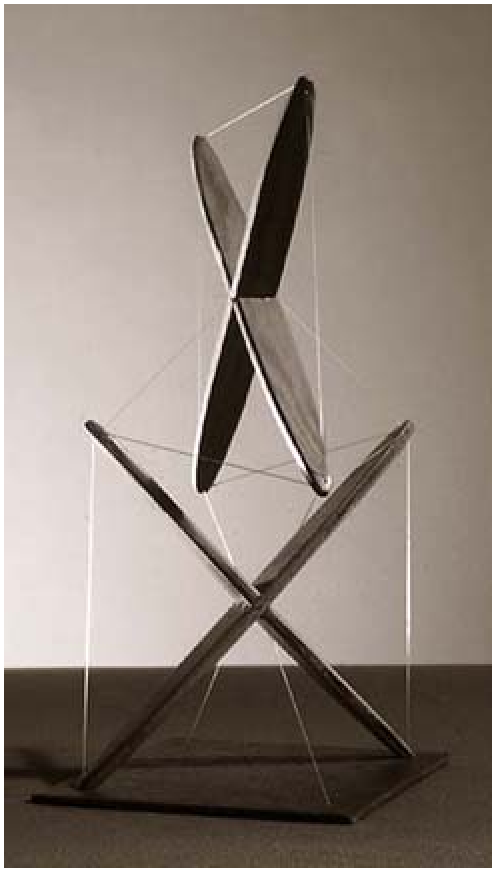


FIGURE 6. X-Piece (Snelson, K., Reproduced with permission).

patent. (see Appendix A.3) Unfortunately Snelson's name appeared infrequently in scientific publications; that spot was reserved for Fuller.

Both Meitner and Snelson had reason to be bitter over their exclusion from the scientific community. Crawford et al. concluded that Meitner's standing in the scientific community was harmed, but Meitner "complained very little, and forgave a great deal" [11]. The same observation could be made of Snelson and tensegrity. In a letter to Motro, Snelson notes:

> . . . I see the richness of the floating compression principle to lie in the way I've used it from the beginning, for no other purpose than to unveil the exquisite beauty of structure itself. Consciously or unconsciously we respond to the many aspects of order in nature. For me, these studies in forces are a rich source for an art which celebrates the aesthetic of structure, of physical forces at work; force-diagrams in three-dimensional space, as I describe them. [12]

In contrast to Meitner and Snelson, both Hahn and Fuller were recognized by the scientific community. After the decimation of Nagasaki and Hiroshima, Hahn became instantly famous in Germany as "the Nobel laureate, the decent German who was not a Nazi, the pure scientist who had discovered nuclear fission but never worked on a bomb" [11]. While "the perception and history of the discovery [of fission] has been skewed by the one-sided award to Hahn" [11], the same may be noted, to a much lesser degree, of the recognition and admiration Fuller enjoyed with his geodesic domes, such as the one found in the U.S. Pavilion for the World's Fair in Canada.

In the end Snelson dedicated much of his career to the design and the assembly of tensegrity structures. Fuller, on the other hand, placed an "emphasis on geodesic domes rather than tensegrity structures" [13]. The scientific community, regardless of the public's or Fuller's perception, does not regard geodesic domes as tensegrity structures.

History, however, has corrected itself. In a 2004 article in *Science Week*, Hahn's undeserving credit was replaced by Meitner's contribution:

> History has its own balance sheet: Until 1997, element 105 was unofficially known as hahnium. In 1997, the International Union of Pure and Applied Chemistry adopted the name dubnium for element 105 and the name meitnerium for element 109. The element hahnium no longer exists. [14]

In the summer of 2008, the Whitney Museum of American Art had an exhibit dedicated to Buckminster Fuller. In a lone corner display case stood a copy of the first tensegrity model created by Snelson (the X-Piece) and a letter from Fuller to Snelson dated December 22, 1949 that effectively acknowledged Snelson's discovery of tensegrity [12]. To the casual observer the X-Piece may have been one of Fuller's designs. To the Museum's discredit, there was not an accurate description of the X-Piece's history, the letter from Fuller or the significant contribution of both to tensegrity. History's balance sheet, it would seem, is still being filled in.

Patents

Buckminster Fuller Patent

Buckminster Fuller's patent "Tensile-Integrity Structures" (1962) describes a structural system in which " . . . compression elements become small islands in a sea of tension" [15]. Fuller continues his description by making an analogy, it would seem, to suspension bridges and notes that the tensegrity structure would aid in "taking some of the compression out of the 'compression towers' . . . through the creation of a structure having discontinuous compression . . . and continuous tension in wherein the islands of compression in the mast are progressively reduced in individual size & total mass" [15].

Kenneth Snelson Patent

Kenneth Snelson's "Continuous Tension, Discontinuous Compression Structures" patent (1965) states that "a single module may possess the characteristics of having all of the compression members therein isolated from each other by the tension network" [16]. He defines a module as "an arrangement of compression members acting as the 'bones' or

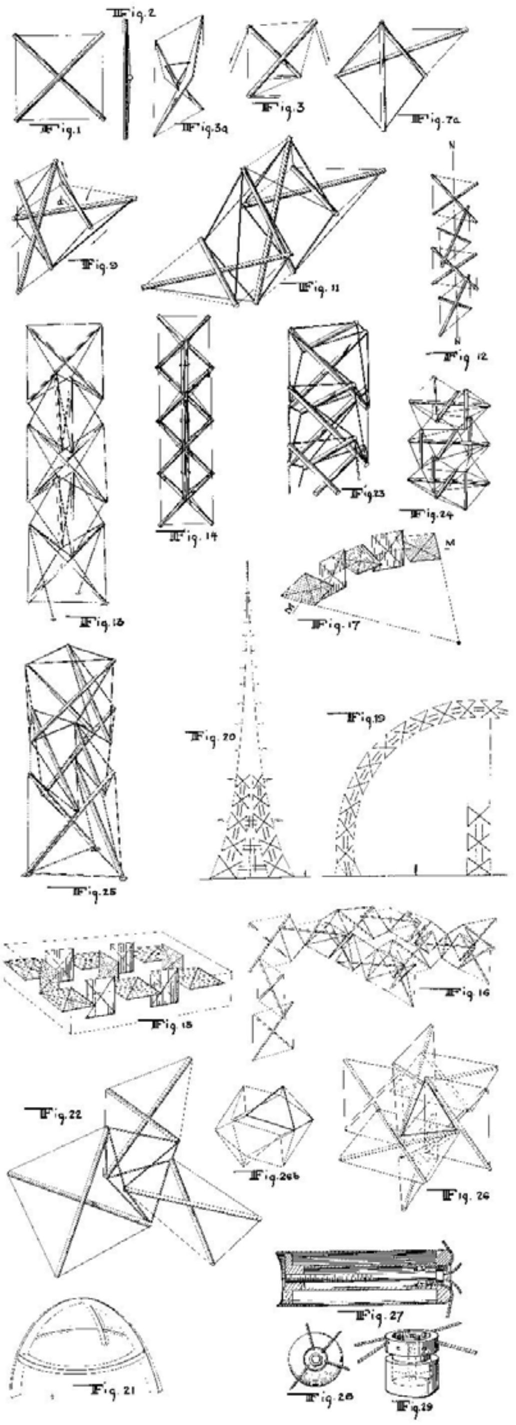


FIGURE 7. Kenneth Snelson’s U.S. Patent 3,169,611: Continuous Tension, Discontinuous Compression Structures (U.S. Patent Office).

skeleton . . . held in relatively rigid relationship to each other by a network of tension members." All of the modules may be used as building blocks and are designed for "discontinuous compression, continuous tension characteristics" [16]. (See Figure 7)

David Georges Emmerich Patent

The patent submitted by David Georges Emmerich, "Contructions de Reseaux Autotendantes" (1963), describes "Autoendante" as a "self-stressing structure consist[ing] of bars and cables assembled in such a way that the bars remain isolated in a continuum of cables. All these elements must be spaced rigidly and at the same time interlocked by the pre-stressing resulting from the internal stressing of cables without the need for extra bearings and anchorage. The whole is maintained firmly like a self-supporting structure, whence the term self-stressing" [17].

Karl Ioganson. R. Burkhardt's "A Practical Guide to Tensegrity Design" discusses the relationship between the work of Karl Ioganson and Emmerich [13]. Burkhardt notes the questionable nature of the claim that the Latvian artist Ioganson displayed a tensegrity prism in Moscow in 1920-21. This prism is known only through photographs because it was demolished by the Soviet regime in the mid-1920s. It is interesting to note, however, that Emmerich based his work on a different structure by Ioganson.

Snelson's letter to Maria Gough, dated June 17, 2003, addressed Ioganson's IX model that was presented by Viacheslav Koleichuk in a 1992 Guggenheim show. Snelson claims that "Koleichuk would have no way of guessing at the object, sticks positioned and strings properly attached, except that he had studied my work, or Bucky Fuller's or David

Emmerich's" [18]. It is unclear, however, if the work was actually recreated based on an unclear photograph or some other work as Burkhardt proposes.

Noteworthy Structures

Widely accepted as the first tensegrity structure, the X-piece was designed and built by Snelson in 1948. Figure 6 is a reproduction of the original. Snelson notes that he had given this to Fuller and that it had subsequently "disappeared" from Fuller's apartment [12].

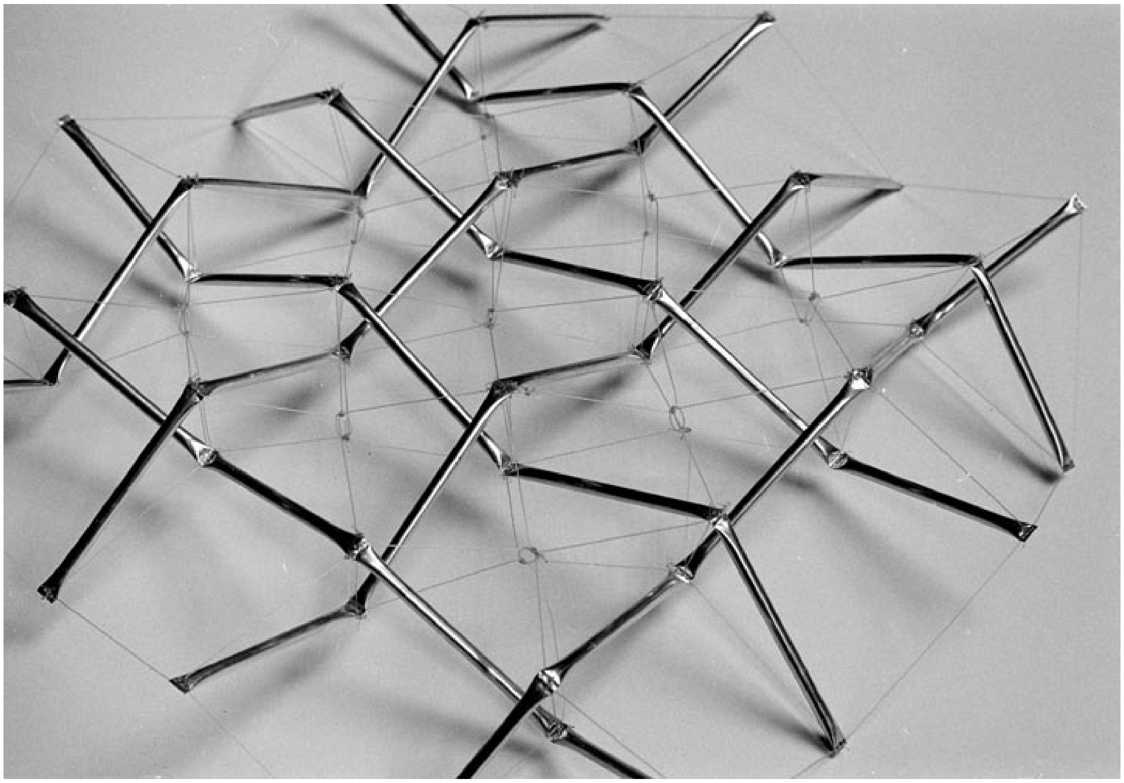


FIGURE 8. Experimental Planar Structures from 1961: Woven Planes (Snelson, K., Reproduced with permission).

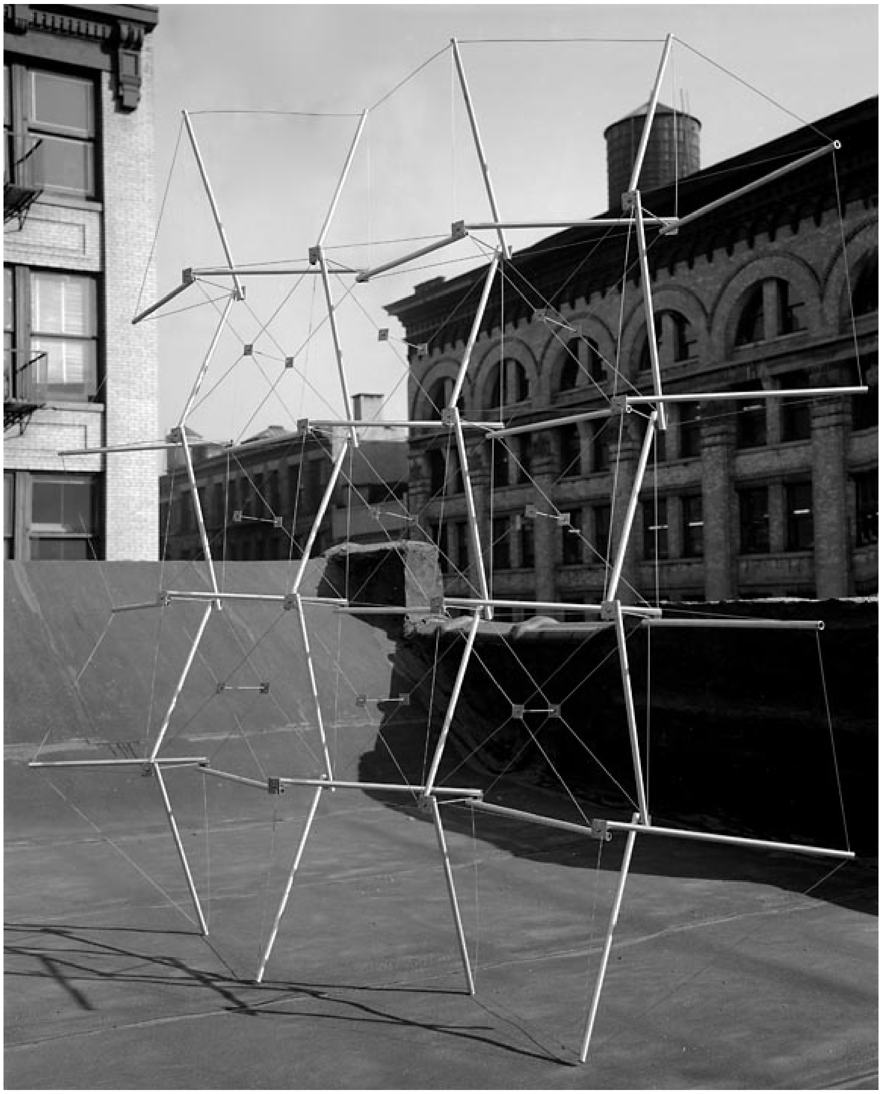


FIGURE 9. Experimental Planar Structures from 1961: Woven Planes on rooftop (Snelson, K., Reproduced with permission).

The first known contiguous tensegrity structures found during research for this thesis were also designed and built by Snelson (see Figure 8 and Figure 9). Snelson labeled them as “Experimental Planar Structures: Woven Planes” (see Appendix A.3). Almost every overview published on the topic of tensegrity has furthermore included reference to

Snelson's "Needle Tower," which was designed and produced in 1968 (see Figure 10 and Figure 11). People look at it in awe, wondering how it supports itself. As a child I looked at similar structures in museums and believed them to be art, rather than a possible new and efficient structural design. Snelson continued his exploration of tensegrity in the contiguous strut Zig-Zag tower, a work that was designed and fabricated in 1997 (see Figure 12).

FIGURE 10. Needle Tower, 1968, 60 x 20 x 20 feet, Collection: Hirshhorn Museum and Sculpture Garden, Washington, D.C. (Snelson, K., Reproduced with permission).

Another noteworthy structure is the Georgia Dome, which provided the inspiration for this thesis. The Georgia Dome is the largest cable-supported domed stadium in the world, seating 71,250 spectators [19]. It is the only cable dome discussed that is spatially triangulated. It is important to note that the tension hoop links the entire tier of the system together, as opposed to a "true" tensegrity system that instead acts individually and loads cascade to the neighboring simplex.

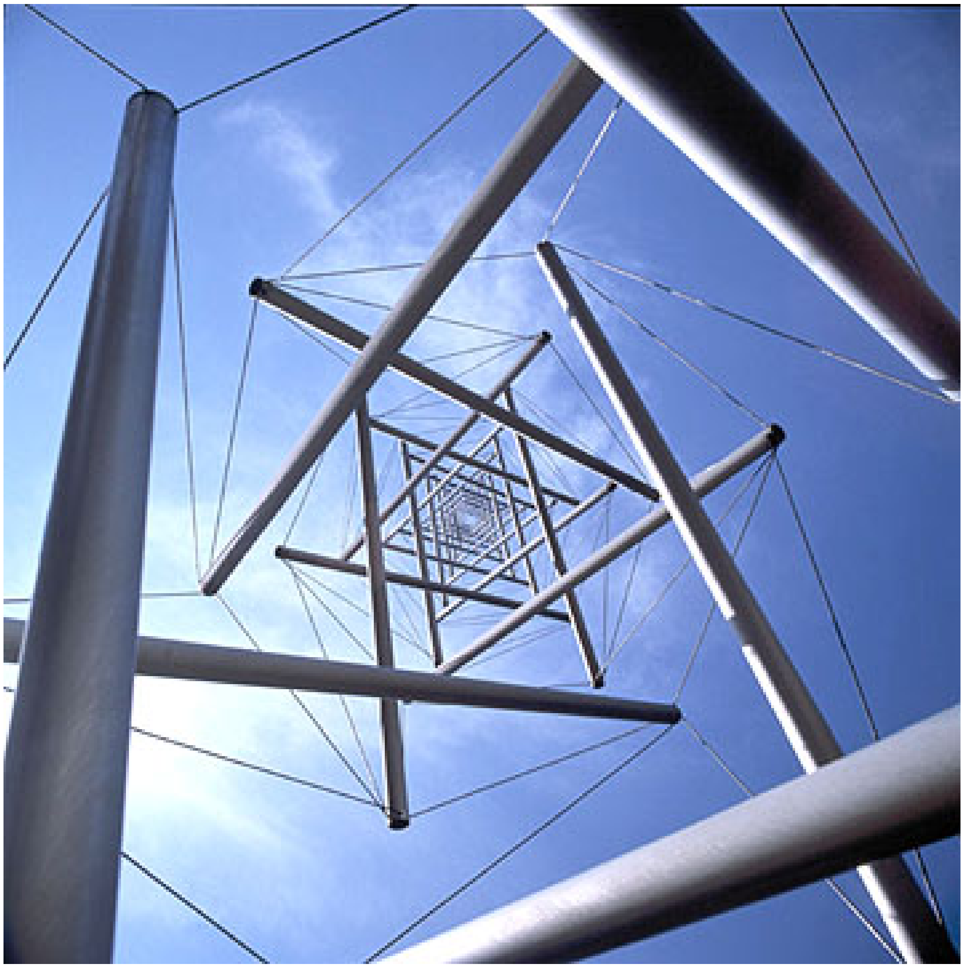


FIGURE 11. Needle Tower, 1968 (Snelson, K., Reproduced with permission),

The tallest tensegrity tower in the world at the time of the publication of this thesis is the Tower at Rostock, which was designed by Mike Schlaich and built by Schlaich Bergermann und Partner in 2003 (see Figure 13). Schlaich notes that "on first sight the structure appears confusing. Even experienced engineers need time to understand the load transfer between the tower components" [20]. The structure is comprised of "two bars in

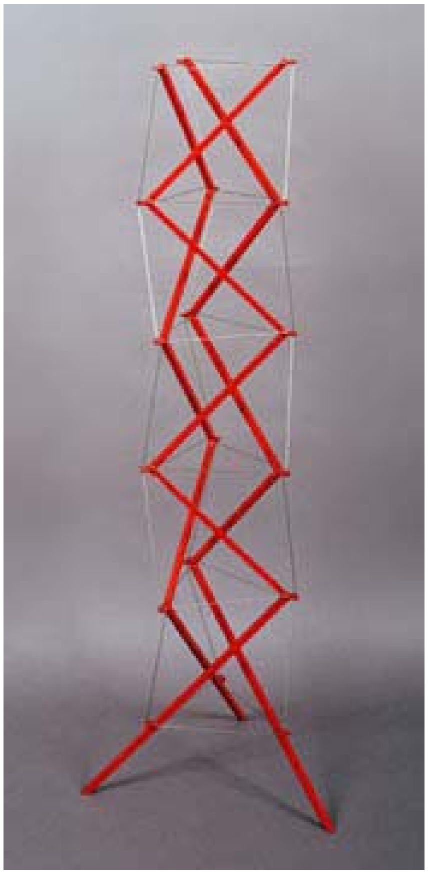


FIGURE 12. Zig-Zag Tower, 1997 painted stainless steel, 45.5" x 9" x 7.75" (Snelson, K., Reproduced with permission).

compression and 4 cables are joined at each node" [20]. Similarities with Snelson's Zig-Zag Tower may explain why Schlaich describes the tower as an "homage to Snelson." (See Appendix A.2) Schlaich notes that "these extremely lightweight and transparent structures require high pre-tensioning for stability" [20]. The drawback to this is that "high pre-tensioning can also reduce the bearing capacity, e.g. highly compressed tubes might buckle earlier" [20].

FIGURE 13. Messeturm in Rostock (tower in Rostock), (Schlaich, M., Reproduced with permission).

At the time the Tower at Rostock was built it was difficult to achieve precise preloads due to the limitations in the cable end fittings. Schlaich notes that "only 20mm is necessary to reach the desired pre-tensioning(1100kN). A variation of only 10mm can decrease the pre-tensioning by up to 50%" [20]. As a result it may be considered an understatement to say that "the tower could only work if very tight tolerances were respected" [20]. Following the construction of the tower, the ability to preload cables has since become a less complex task.

Schlaich notes that there was a concern with rigidity and the use of a tensegrity design for the Tower. However, "after it became clear that the tower would neither support large signboards nor would be climbed by its users, the large deflections of a tensegrity structure were no longer a criteria for exclusion"[20]. The extension of the tower from 30m to 60m involved "using a trick permitting contact of certain compression elements." [20]. Schlaich concludes that "tensegrity towers are extremely flexible and [yield] structures of very limited practical use" [20].

Biological Cell Structure

Donald Ingber, MD, Ph.D., professor and researcher at Children's Hospital and Harvard Medical School, credits Kenneth Snelson's sculpture as inspiration for his life's work in cell structure. In an interview with Public Radio's Studio 360, Ingber recalls viewing Snelson's "elegant" Needle Tower in 1975 as an undergraduate, and the way it reacted to stimuli, which occurred when he knocked it. He notes that he was inspired to pursue tensegrity and later to identify its use in organizing cells through the cytoskeleton [21]. Merriam-Webster's Medical Dictionary defines the cytoskeleton (CSK) as the "network of protein filaments and microtubules in the cytoplasm that controls cell shape, maintains intracellular organization, and is involved in cell movement" [22].

Ingber notes that “this relatively simple theory [tensegrity] can explain much of the complexity of pattern and structure that is observed within the cytoskeleton (CSK) of living cells.” The advantage of tensegrity is its ability to “sense and immediately respond to physical stimuli from both inside and outside the cell” [23]. This proves D’arcy Thompson’s assertion that cells, although complex, may be “governed by simple rules” [24]. Most importantly, Ingber notes that “understanding cell behavior . . . has led to a better understanding of diseases that strike down tissue architecture, like cancer. . . . tensegrity will probably help scientists better understand asthma, emphysema, hypertension, and osteoporosis, as well as how life first originated on Earth” [25].

Balloon and Spring Mattress Analogy

One of the most succinct descriptions of tensegrity that also utilizes a common household item is Motro’s balloon analogy. Motro notes that “a balloon can be considered as a tensegrity system since it is a stable self-balancing system made up of two components: a compressed component, the air and a tensioned component, the membrane“ [26]. Continuing with the tensegrity analogy, Motro relates a spring mattress to a bi-directional tensegrity grid.

Motro notes that a spring mattress exhibits a “similar external behaviour and internal layout (‘islands of compression in an ocean of tension’).” There are essentially four different aspects of tensegrity described. The first aspect is that the exterior of the structure has a border or the top, bottom and sides of the mattress. The second aspect is the flexibility of the border surfaces; this flexibility is similar to cables that slacken for tensegrity. The third is the grid that is contained within the mattress. The grid is a simplex of cables and struts similar to that in Figure 29 [26].

Cable Dome

Non-Contiguous Grids

Cable domes, with their discontinuously located struts, may be considered non-contiguous strut grids that generally contain " . . . large internal forces, very low stiffness and heavy weight and are actually sensitive to support positions" [27]. Wang classifies the cable dome as being less efficient than the alternative contiguous cables. He notes that the weakness of the cable dome is the cable to strut connection between a simplex, which results in an indirect transfer of load through the joint to the cable [27]. In short, Wang claims free standing tensegrity structures are inefficient because of Fuller's patent definition that describes "islands of compression in a sea of tension" [27].

Part of Wang's rationale is the fact that cable strut systems contain "no boundary anchoring system" and they contain continuous cables with free standing, or unrestrained, pin jointed struts. This does not, however, mean cable domes are heavier. To clarify, "cable domes are lighter but are actually not highly structurally efficient whose weight reduction is due to high strength of cables. In comparison, cable-strut grids save a boundary ring beam and avoid [a] complicated construction process" [27].

Geometry

Analysis performed by Gerardo Castro and M. Levy on the Georgia Dome suggests that increased post height equates to lower cost (see Figures 14 and 15). Their analysis also indicates that "a two ring configuration is more economical than the three ring" as-built configuration [19]. The cable dome support structure that was modeled for this thesis incorporates a two ring configuration with increased post height for this reason.

At the bottom of each ring in the cable dome is a hoop cable. The cable is tied to other struts and acts to restrain their base movement to the degree that self and prestress allow. As Campbell notes, under concentrating loading hoop cables act to diminish and dissipate the stress imparted to the structural members. This also, however, results in a relatively large tensional load in the hoops, while ensuring that the overall structure is rigid [28].

The Crown Colliseum in Fayetteville, North Carolina is a cable dome in which "the instability encountered in the preliminary design occurred at the bottom of the outer mast . . . due to the fact the ends of outer diagonals were located above the top elevation of the outer struts" [29]. For this reason the thesis cable dome outer diagonals are located below the top of the elevation of the outer struts. In addition, Campbell explains that "most Cabledomes have been built with span to rise ratios greater than 12" [30]. In this thesis, the ratio is modeled at approximately 8.9 (See Figure 27).

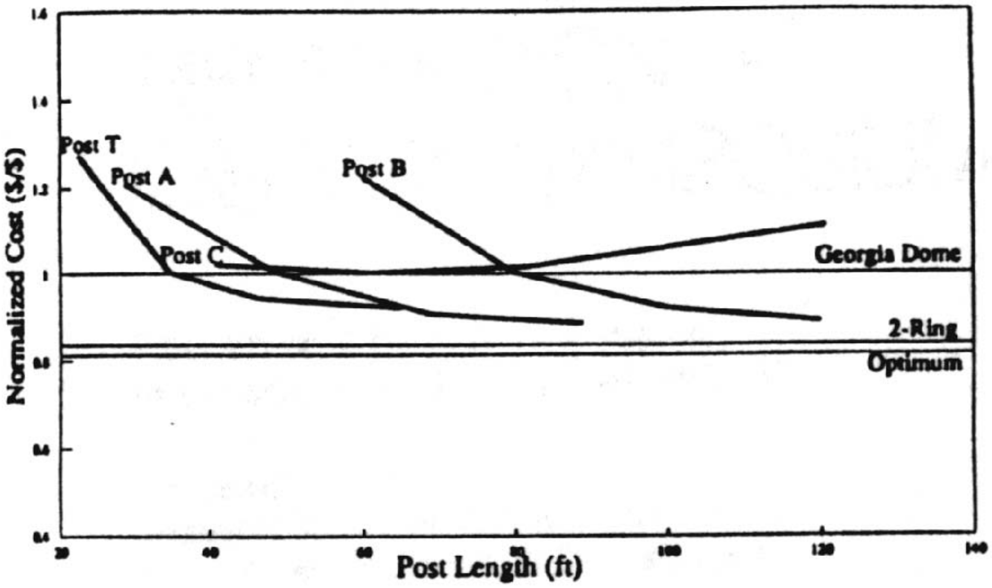


FIGURE 14. Georgia Dome Cost vs. Post Length (Castro, G. and Levy, M. P., 1992, "Analysis of the Georgia Dome Cable Roof," Proceedings of the Eighth Conference of Computing in Civil Engineering and Geographic Information Systems Symposium, ASCE, Dallas, TX, Figure 9, Reproduced with permission).

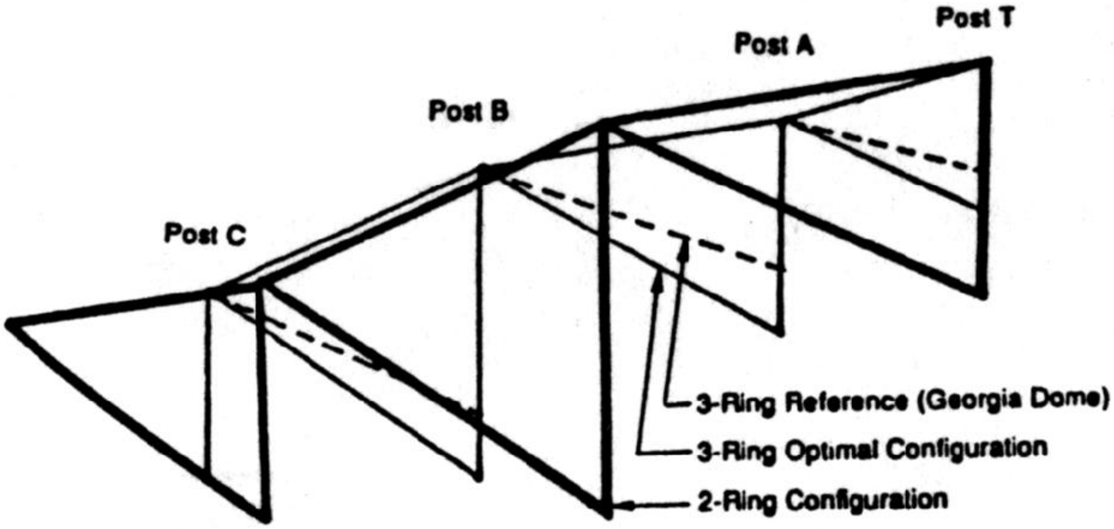


FIGURE 15. Georgia Dome Optimal Configuration (Castro, G. and Levy, M. P., 1992, "Analysis of the Georgia Dome Cable Roof," Proceedings of the Eighth Conference of Computing in Civil Engineering and Geographic Information Systems Symposium, ASCE, Dallas, TX, Figure 10, Reproduced with permission).

Cable dome not tensegrity? Ariel Hanaor describes that a cable dome is neither a dome nor a tensegrity structure. Instead, he notes that it is “a straightforward suspended cable structure, where struts simply serve as spacers between the supporting cables and the supported dome-shaped (but not dome acting) upper surface.” (see Appendix A.1) Motro concurs, stating that the compression ring is on the exterior of the system and not the interior. However, he also notes that “it is obvious that these cable-domes are very efficient” [26]. For the same reason as Motro, Snelson explains that these particular types of domes “can not be considered tensegrity . . . .they are, essentially, bicycle wheels” [18]. The compression ring in the bicycle wheel is in the rim itself, in the exterior of the grid.

Cable Dome Pre-Load/Prestress

Prestress. For the infancy of cable domes it was thought appropriate to prestress the cable at 20% of the ultimate tensile strength to achieve maximum stiffness [28]. “The advantages in construction of lower prestress are obvious . . . less prestress directly equates to less work. As geometric stiffness is reduced, greater deformation is required for the structure to resist a given load . . . this generally means a larger portion of the structure is engaged in resisting a given load distribution” [28]. For a more rigid structure the local loads are resisted universally instead of locally, resulting in an advantage for non-symmetrically loaded structures or other upset modes for a given system.

Gunnar Tibert notes that as a result of cable relaxation “the magnitude of the pretensioning force varies from structure to structure, but must, due to stress relaxation, not be greater than 45% of the breaking force of the cable . . . “ [31]. Testing resulted in permanent deformation of steel wires preloaded greater than 50% of their ultimate tensile strength and that preload, or prestress, should not be greater than 45% of ultimate tensile strength [31]. This rule of thumb was utilized in those wires modeled and preloaded for this thesis.

Self-stress. Motro notes that the designer must choose his self-stress and prestress carefully. He describes that “the range of pre- or self-stress shapes is directly related to the number of restrictive conditions imposed by the designer.” As a result, “designers have to solve a very specific problem related to the implementation of self-stress” [26]. Campbell explains that prestress for most cable domes is very small, and that the load of the structure imparts the “majority of the hoop tension” in cabledomes [28]. Motro concludes that self stress is a “key feature of tensegrity systems. It must be studied with special care not only to make an optimum choice of the initial state, but also in accordance with practical aspects for implementation monitoring” [26].

Triangulation vs. Radial Configuration

David Campbell’s paper entitled “Effects of Spatial Triangulation on the Behavior of ‘Tensegrity’ Domes” compares circular, 394 ft. span, spatially triangulated and radial oriented dome structures, each with a dead load of 6.6 lb/ft.^{2} [30]. As a reference, the approximate dead load of the proposed cable dome in this thesis is 8 lb/ft.^{2} A triangulated structure utilizes cables that run diagonally to their support struts (see Figure 24), as opposed to a radial configuration that aligns the cable perpendicular to the attaching structure.

Campbell concludes that, “generally, this added complexity [from triangulation] does not seem to yield any direct benefits other than a somewhat increased stiffness in response to load concentrations. . . . The cabledome generally exhibits greater stiffness, much reduced to non-uniform and concentrated loads, an insensitivity to fabrication errors, as well as greater design flexibility of roof form than the triangulated dome system" [30]. Unfortunately, for an application that is required to see potential point (concentrated) loading and, at the same time, is required to see reverse (-Z) loading with maximum stiffness (minimal deflection), the same conclusion cannot be drawn. For a more detailed review of this paper see Appendix B. It is for this reason that a triangulated tensegrity structure is utilized for the loading conditions in the Space Shuttle. Campbell concurs: "I would be surprised if the radial non-triangulated Cabledome could be adapted reasonably to the configuration(s) you are working with. Triangulation of the network would no doubt be useful as would adoption of the double layer tensegrity grid." (see Appendix A.4)

Cable Domes Around the World

The popularity of cable domes is clearly evident in their world-wide construction. Cable domes that have been built with membrane roofs include the Seoul 1986 Olympics domes, S. Korea Gymnastics Arena (393 ft. span, 15k seats) and Fencing Arena (305 ft. span, 7k seats), Redbird Arena in Illinois (10k seats, 1988), Tropicana Field in St. Petersburg (1988), Georgia Dome in Atlanta (1992), and Tayouan Arena (447 ft span, 15k seats) in Taiwan, Republic of China (1993). The Crown Coliseum in North Carolina (330 ft span, 13k seats), built in 1997, is a cable dome that contains a rigid panel roof.

Contiguous Grid

Contiguous Struts

Contiguous struts seem to offer the most promise for delivering a rigid tensegrity structure suitable for use on the Space Shuttle. Various works that support the struts' ability to add rigidity to a structure include V.G. Jauregui's thesis, Motro's and Wang's extensive work, discussions with Ariel Hanaor, and a review of Hanaor's latest paper, "The Concept of Structural Depth as Applied to Certain Bar-tendon Assemblies." Wang summarizes the value of contiguous struts by stating that "contiguous strut tensegrity grids present much better structural efficiency over non-contiguous strut tensegrity grids." Therefore, according to Wang, an efficient structure should be based on contiguous grids [27].

Motro takes exception to the distinction between contiguous and non contiguous systems. He asserts that a chain of struts can be considered one solitary, compression member and thus does not require a special classification for contiguous grids [26]. This would ensure inclusion within the tensegrity domain; however, some definitions of tensegrity identify the end of the compressed element, or locations where cables are attached, as the node.

Classes of tensegrity structures have been defined to distinguish the level of contact that one compressive member has with another. For example, a "class k tensegrity structure for k > 1 allows k compressive members to be connected in a ball joint (so as not to apply torque from one member to another)" [32]. A non-contiguous grid would be of the order of k=1. Contiguous struts are equal to a tensegrity structure of the order of k=2, where the struts are permitted to be in contact with each other. As noted previously "such a structure does not comply with the definition of tensegrity proposed by Pugh" [9]. In addition, Wang states that the "resulting structural weight of most grids can be lighter than space trusses" [27].

Isolation of Struts in Grid

Wang notes that because there is an inefficient load transfer at the joints in non-contiguous strut grids there is a resultant increase in the cable. This is primarily a result of "infinitesimal mechanisms (or near-mechanism geometry) . . . resulting in much-reduced resistant lever arm and low-stiffness." The largest contributing factor is the isolation of the struts in the grid. In summary, Wang concludes that "design results show that [a] non-contiguous strut grid is much larger in internal forces, weight and deflection than contiguous strut grids, so are contiguous strut grids than the space truss except for the deflection aspect due to different material application" [27].

Properties of Contiguous Strut Tensegrity Grids

Gaps or the "shelf" in the case of the thesis cable dome models mean that contiguous struts may not be the most efficient choice for an opening configuration. As Wang notes "contiguous strut configuration with openings (or called 'plane-filling forms') are of low structural efficiency owing to the resulting isolation of struts, which results in cables sustaining tension in the compressive layer" [27]. This is a predicament since we desire rigidity and efficiency, both of which held promise in contiguous grids. In addition, internal loads in contiguous grids are greater than the traditional space truss, resulting in a tensegrity grid that is "40% heavier than that of the space truss" [27].

Pre-stress and Preload

Ariel Hanaor classifies two different classes of tensegrity structures. He describes class I as "geometrically rigid and statically indeterminate structures" while class II "are statically and kinematically indeterminate structures with infinitesimal mechanisms." [33] Prestress applied to both class I and II structure results in either improvement of the design or "geometric integrity" [33].

Hanaor also notes that prestress is useful for improving stiffness it is not a viable means for increasing efficiency [34]. Wang agrees that stability is not determined by prestress, and it is not an indespensible tool. However, Wang does clarify that the distribution of internal forces is more uniform and typically stress and deflection are low in geometrically rigid structures [27].

Motro discusses preload and his attempts to streamline the tensegrity design process. He notes that studying Snelson's structure is essential because all of Snelson's structures were prestressed, but at the same time noted that it was not possible to extract generalized preload procedures from the process [26]. The addition of prestress effectively reduces the design steps to finding self-stress coefficient values, solving the linear homogeneous system of equilibrium equations and identification of the form with additional design iterations required [26]. For the thesis the preferred method is to prestress the structure to 45% of its ultimate tensile strength (see Cable Dome prestress discuss in the beginning of this chapter). In addition, the structure will be loaded and sized to achieve a "geometrically rigid" structure with the "appropriate selection of topology and geometry" [35].

Efficiency

Maurıcio de Oliveira et al. describe the perfect world tensegrity system in which non-contiguous, cabledomes "never experienced torque nor reversal in load direction, allowing efficiency and the choice of materials. The entire structure can bend yet no bending moments are applied to any structural member." They further note the main theoretical advantage to a tensegrity system in which highly efficient cables relieve struts of their compressive load; therefore "by using more strings, tensegrity structure design can save mass" [36].

Along the same lines Juan also notes that "structural material is only needed in the load paths, so tensegrity structures, by carefully placing the compression elements, are capable of increasing the resistance/weight ratio of traditional structures" [9]. Because tensegrity structures are not just materials but instead contain mechanisms, it is doubtful that tensegrity applies.

All of the above theory sounds attractive, however theoretically perfect qualities for a suspension system may not be attainable. Hanaor states that "tensegrity structures as spanning structures (such as free-standing domes or planar grids) . . . are inherently less efficient than conventional bar structures, due to the reduced effective structural depth. As top cables go slack structural depth is in effect halved." (see Appendix A.1)

Tensegrity Weight and Rigidity and Sizing (EA Ratio)

Wang summarizes Hanaor's study of a flat tensegrity layout based on the triangulated simplexes: "the self-weight of the geometrically rigid tensegrity grid is nearly twice that of the studied space grid." He notes that extended bars are "the reason for the heavy weight of the tensegrity grids" [27]. Alternatively, the bars could be made shorter. However, as Snelson explains, "short compression struts mean long tension lines which mean extreme elasticity. The struts can't be all that lightweight because they must support enormous compression loads. They need heavy and robust end-fixtures in order to absorb the powerful tension forces that pull outwardly with great cumulative force" [17]. Motro states that "for sufficient rigidity, our experience in this field has shown that a rigidity ratio (EAstruts/EAcables) close to 10 is satisfactory. Above this, the behaviour is too flexible and leads to over sizing the cable elements. Below 10, the struts are overloaded and thus oversized" [26].

Structural Efficiency Ratio

Hanaor uses a structural efficiency ratio to classify systems of tensegrity structures. The ratio is "defined as the ratio of the load bearing capacity of the structure to its weight.." [34]. Hanaor notes that two variables, load and material type, must be taken into consideration when comparing structural efficiency ratios: " . . . the structural efficiency ratios of structures of similar type and geometry tend to be higher the more heavily the structure is loaded, even though the actual weight is larger" [34]. He continues to explain that it "is obvious that a structure made of aluminum, for instance, would be lighter than the same structure, subjected to the same load, but made of steel" [34]. However, even when comparing structure designed from the same material, the ratio of cables to compressive bars plays a large role in dictating structural efficiency.

Wang uses a different method. He defines efficiency by the "reverse of the weight of the grid specified to be capable of sustaining the prescribed loading conditions and satisfying service requirements." He notes that for his system "the higher the weight, the lower is the structural efficiency." The key, he explains, is stiffness and minimum deflection. "A structure of low stiffness requires high prestress to meet service requirements, thus internal forces and consequently, self weight is increased" [27].

Hanaor also utilizes structural depth, usually at mid-span, as a tool for assessing structural systems. He notes that "structural depth at a cross-section through the structure, is defined as the lever arm of the resultant internal force couple at the cross-section, balancing the overturning moment produced by the external load on a free body bound by the cross section in question" [34].

Figure16, Figure 17 and Figure 18 all illustrate the point that the longer the span the less efficient the structure. Note that in Figure 16 structures 5 and 6 are the only structures that are built; the rest currently exist only as a paper design. Finally "bar-tendon" assemblies are shown in Figure 19. The systems shown are based on simplexes as previously discussed in this thesis. Hanaor observes that "it should be borne in mind that the design strength of cables is 2.5-3 times that of the bars. The weight of cables ranges from ca. 15% in tensegrity and ATP grids . . . to 20-25% in continuous chord grids (RP, CP)" [34].

<u>Definition of CP, ATP and RP simplexes</u>. CP, ATP and RP simplexes are shown in Figure 20. All of the structures have a continuous bar chord as the compressive component. The structures purpose "is to replace tensile members with tendons to reduce the lengths of compressive bars, thus achieving high structural efficiency" [34]. Those configurations shown were built for gravity loading and as a result when uplift is applied the structural

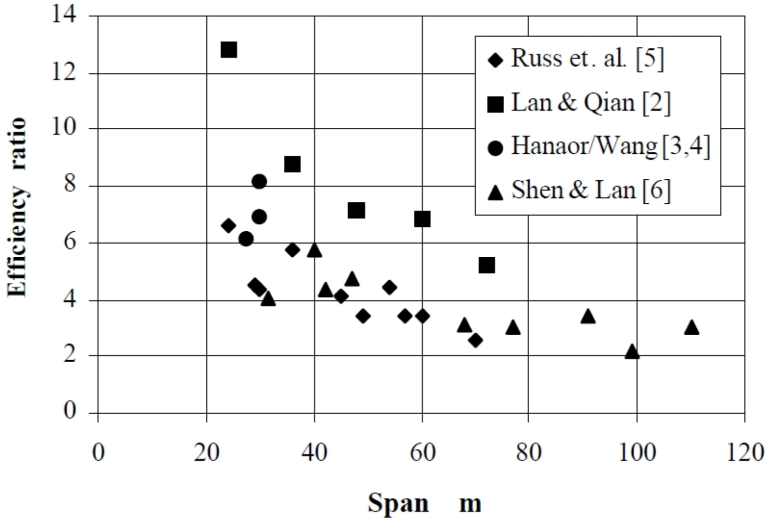


FIGURE 16. Efficiency ratio vs. span of double layer space trusses, adjusted for imposed load of 100 kg/m^2 (Hanaor, A., 2002, "The Concept of Structural Depth as Applied to Certain Bar-tendon Assemblies", Space Structures 5, Proc. 5th International Conference on Space Structures, University of Surrey, Guildford, UK, 19-21, GAR Parke and P. Disney, Eds., Thomas Telford, London, Figure 1 (p.3), Reproduced with permission).

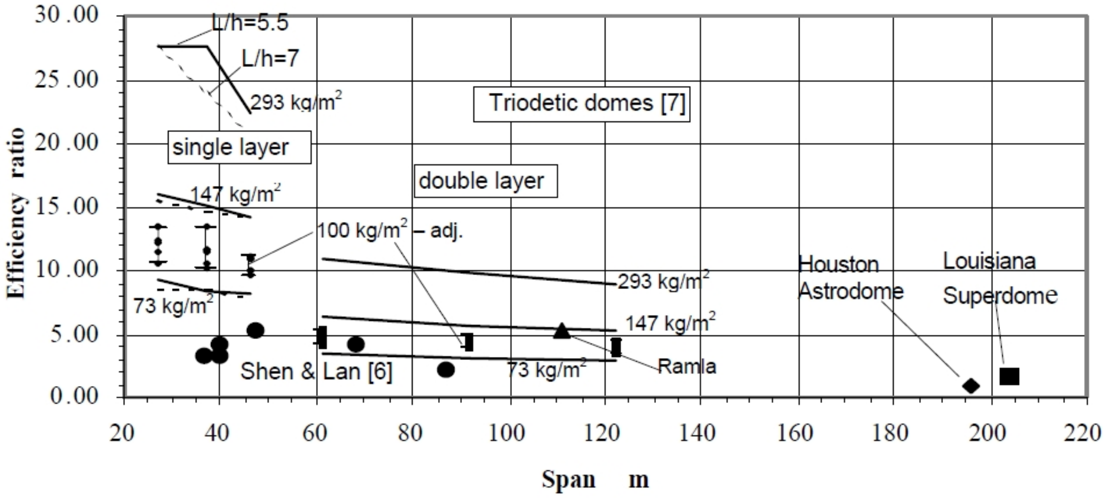


FIGURE 17. Efficiency ratio vs. span for braced domes (Hanaor, A., 2002, "The Concept of Structural Depth as Applied to Certain Bar-tendon Assemblies", Space Structures 5, Proc. 5th International Conference on Space Structures, University of Surrey, Guildford, UK, 19-21, GAR Parke and P. Disney, Eds., Thomas Telford, London, Figure 2 (p.3), Reproduced with permission).

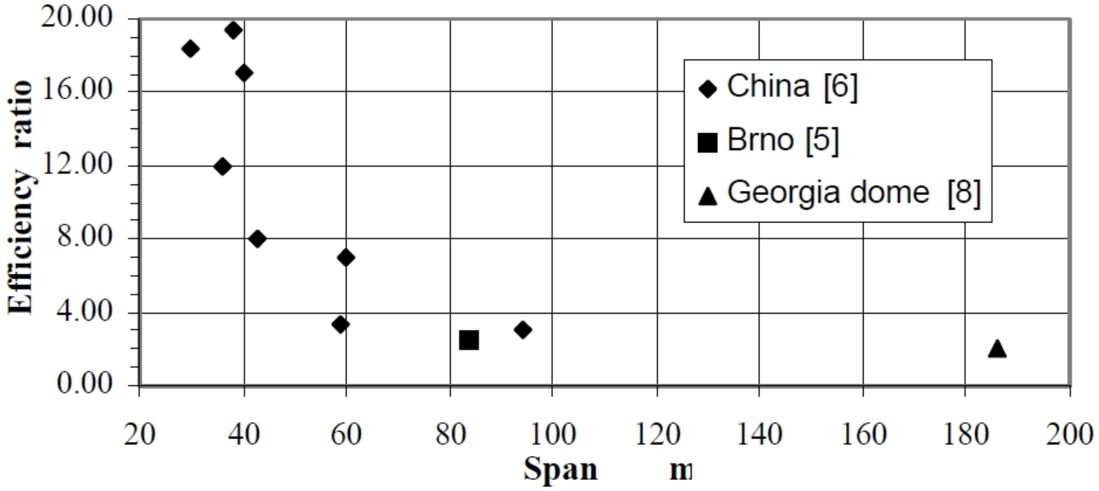


FIGURE 18. Structural efficiency of constructed cable roofs and domes (Hanaor, A. , 2002, "The Concept of Structural Depth as Applied to Certain Bar-tendon Assemblies", Space Structures 5, Proc. 5th International Conference on Space Structures, University of Surrey, Guildford, UK, 19-21 GAR Parke and P Disney, Eds., Thomas Telford, London, Figure 3 (p.4), Reproduced with permission).

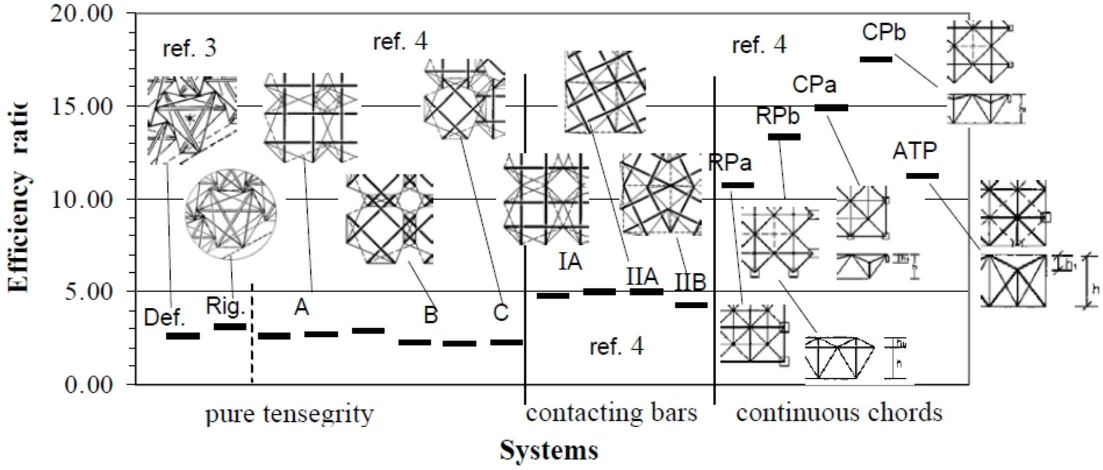


FIGURE 19. Structural efficiency of designed bar-tendon double-layer grids adjusted for imposed load of 100 kg/m^2. Span = 27-30 m. (Hanaor, A. , 2002, "The Concept of Structural Depth as Applied to Certain Bar-tendon Assemblies", Space Structures 5, Proc. 5th International Conference on Space Structures, University of Surrey, Guildford, UK, 19-21 GAR Parke and P Disney, Eds., Thomas Telford, London, Figure 4 (p.4), Reproduced with permission).

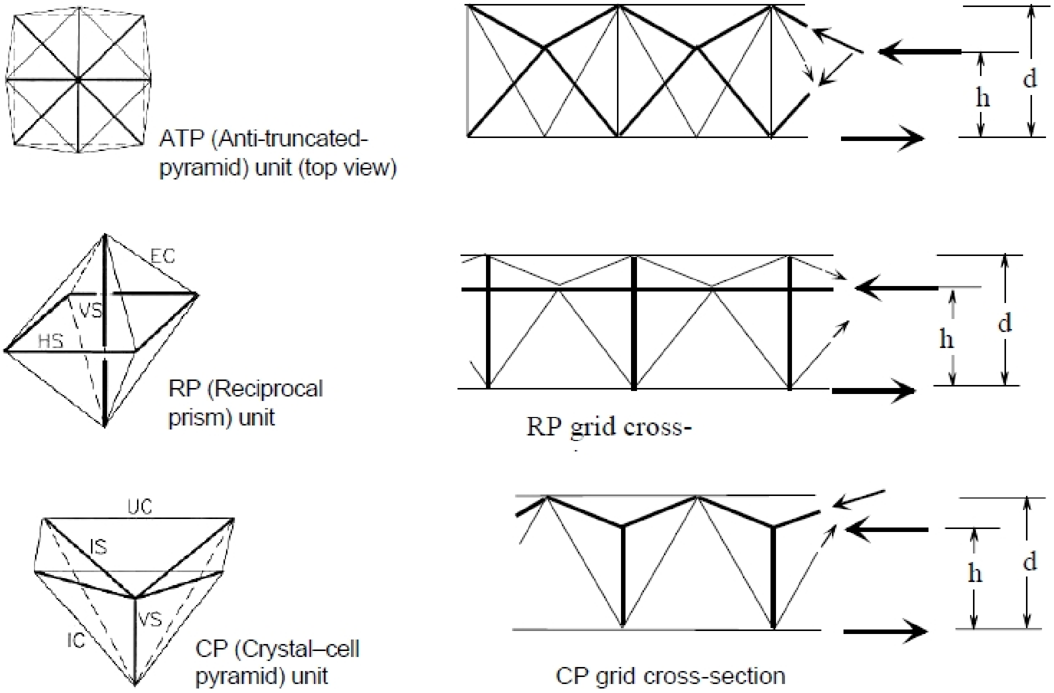


FIGURE 20. Bar-tendon grids with continuous bar chords proposed by Wang (Hanaor, A., 2002, "The Concept of Structural Depth as Applied to Certain Bar-tendon Assemblies", Space Structures 5, Proc. 5th International Conference on Space Structures, University of Surrey, Guildford, UK, 19-21, GAR Parke and P. Disney, Eds., Thomas Telford, London, Figure 12 (p.9), Reproduced with permission).

depth is substantially reduced and “in the case of CP grids vanishes” [34]. Wang has noted lower cables could be added. Wang has suggested that “when the design of the uplift load is not much larger than the downward load, the bottom layer may be attached to lateral supports by cables” [20]. (see Figure 21). This would be a valid approach to take while also decreasing the ratio of the straight strut length to diagonal length. Wang also notes that, “the CP grids save strut weight mostly and the gross weight savings is nearly half compared with space grids” [27]. A use of the CPb grid, or as some call it “diamondshaped tensegrity," was an option for this thesis, however the fact that the grid is ineffective under uplift (reverse g-loading) is worrisome (see Figure 22).

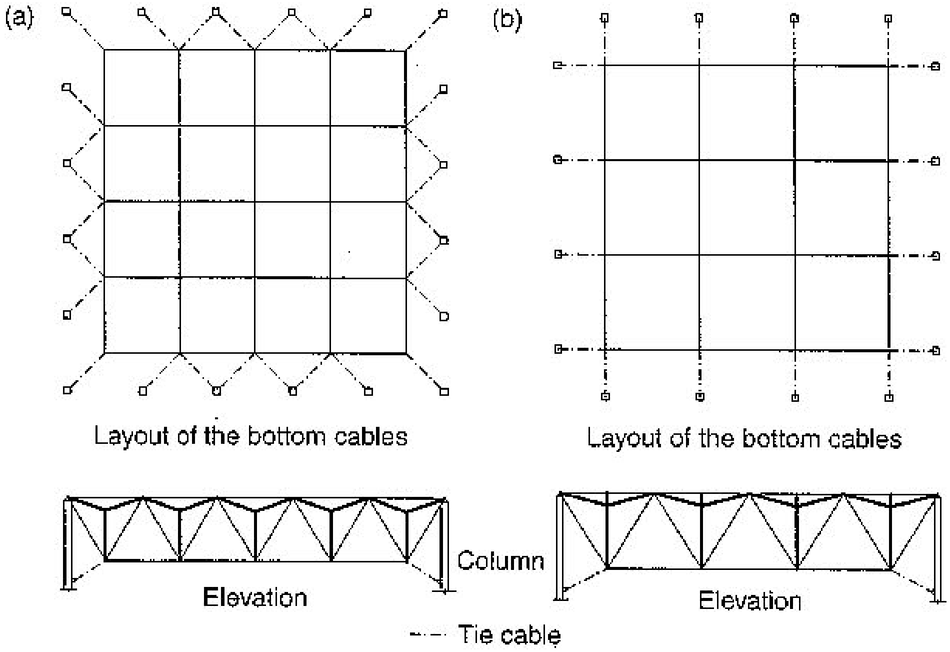


FIGURE 21. Stabilized form of the CP grids: (a) CP-a grid; (b) CP-b grid. (Wang, B.B., 2004, Free Standing Tension Structures, Spon Press NY, NY, Figure 5.11 (p.115), Reproduced with permission)

However, Hanaor notes "some optimization of the relative structural depths for gravity and uplift loads can be performed, but it is doubtful if the result would be an improved structural efficiency compared to conventional double-chord bar grids (at least when material efficiency is factored out)" [34]. A CP grid was not modeled in this thesis for this reason. Wang's summary of efficiency "tensegrity grids are not structurally efficient despite that high-strength cables are introduces as tensional material and that all bars are in compression as they do not comply with the dominant load-transfer pattern."

He believes they are suitable for small spans, for “special architectural requirements . . . or in special functions like deployment” [27].

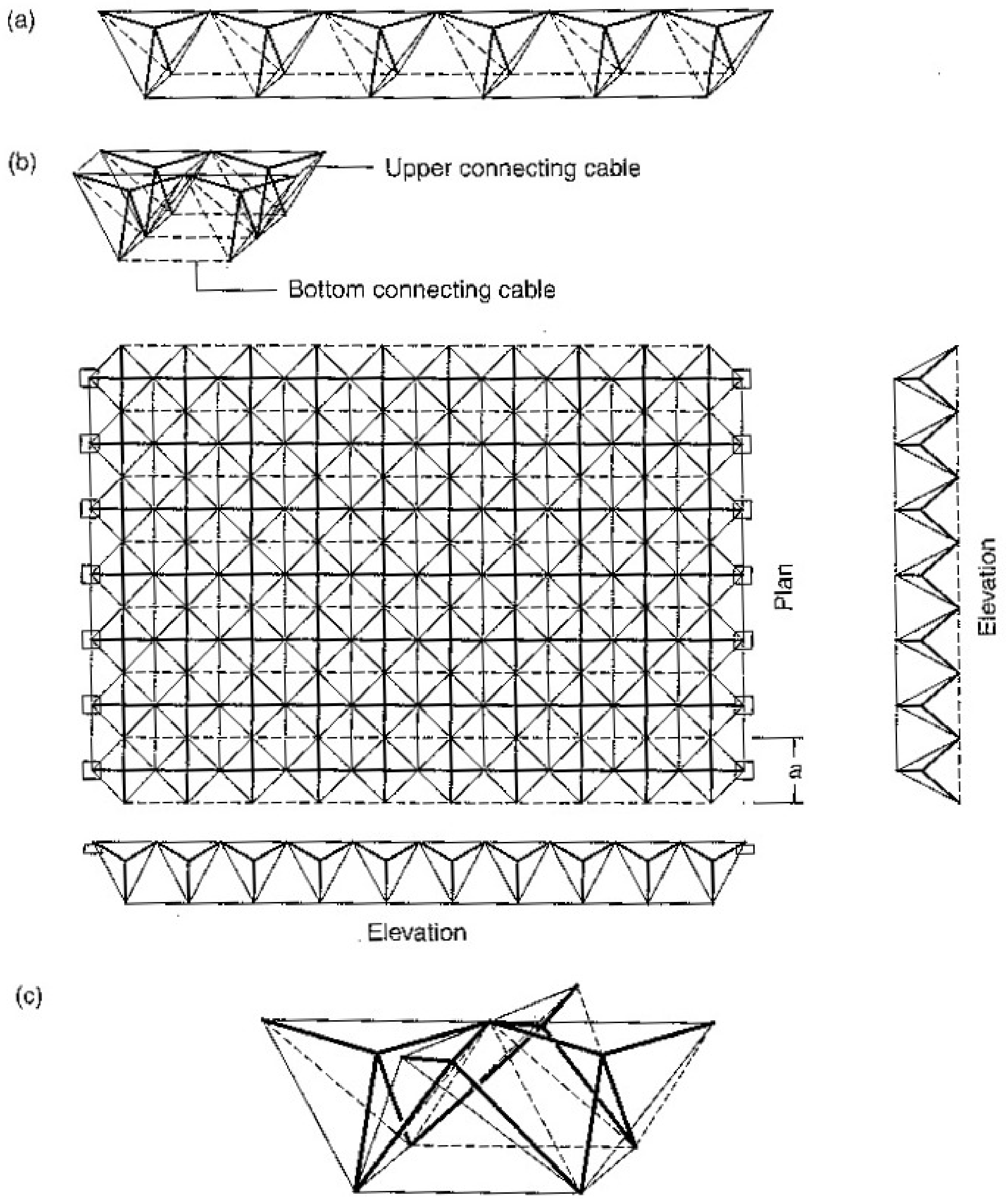


FIGURE 22. Special CP configurations by connecting edges: (a) a CP truss; (b) CP grid; (c) two-way configuration of (a). (Wang, B.B., 2004, Free Standing Tension Structures, Spon Press NY, NY, Figure 7.15 (p.183), Reproduced with permission)

Technological Advancements

Computers

Invariably if analysis is done in any way it involves FEM/FEA. Analysis of a non-contiguous, k=1 tensegrity structure “has been wholly dependent upon the use of digital computing.” [28] Form finding algorithms available include software that performs the force density method. Campbell notes similar programs; for example, “Birdair Inc. successfully employes their matrix analysis algorithm for form finding . . . Another method . . . is the method of dynamics relaxation with Kinetic Damping . . . used by FTL associates” [28]. (See Figure 23)

Materials

A cable material used commonly in aircraft control cables and bridges is 17-4PH steel. Compared to the 6Al-4V baseline titanium strut design the 17-4PH material is (.282lb/in^3 - .160lb/in^3)/.160lb/in^3 = 76.25% heavier and only (168ksi -138ksi)/138ks i= 21.7% stronger. Higher strength steel cables are currently available and marketed as “high strength,” such as Sandvik CS-9A carbon steel wire. Compared to the 6Al-4V baseline titanium strut design the Sandvik CS-9A high strength steel cables are (.282 lb/in^3 - .160lb/in^3)/.160lb/in^3 = 76.25% heavier and (257ksi -138ksi)/138ksi = 86.23% stronger than 6AL-4V Titanium.

One drawback of the 17-4PH, or high strength steel cables in general, is weight. Xin Wang and Zhishen Wu note that steel cables that experience a “sag” as a result of weight and initial cable stress contribute to an overall reduction in the equivalent modulus

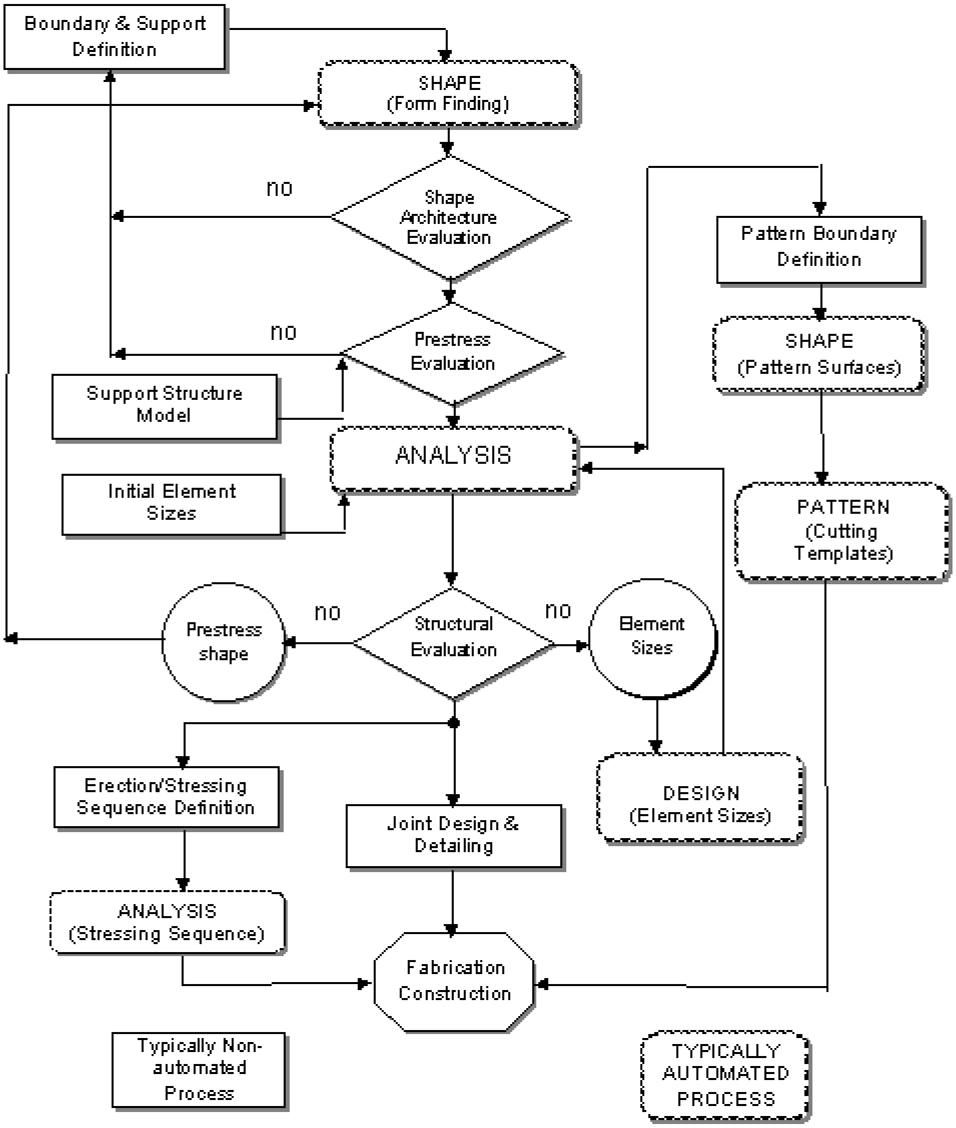


FIGURE 23. Flowchart Illustrating General Approach to Tensile Membrane Structure Design and Engineering (Campbell, D., "The Unique Role of Computing in the Design and Construction of Tensile Membrane Structures," http://www.geigerengineers.com, Figure 1, Reproduced with permission).

that "will decrease with the elongation of the main span, which results in a weakening of the entire bridge stiffness, making the structure more flexible" [37].

One alternative is carbon fiber reinforced polymer (CFRP)/ carbon fiber composite cable (CFCC^{TM}) strands which are (.0710lb/in^3 -.160lb/in^3)/.160lb/in^3 = 55.63% lighter and (312ksi -138ksi)/138ksi = 55.63% stronger than 17-4PH high strength steel. Tokyo Rope Mfg. Co., Ltd produces the CFCC^{TM} strand, which is CFRP, that was installed in June 2007 for testing in the Penobscot Narrows Cable Stayed Bridge in Maine.[38] Specific certification data obtained from the Penobscot bridge CFCC^{TM} strand is utilized in this thesis and the material properties are listed and compared with conventional high strength cable in Table 10.

TABLE 10. Cable Material Properties

Type of Cable	Density (lb/in^3)	Elastic Modulus (ksi)	Tensile Strength (ksi)
17-4PH	.282	28600	168
HS Steel Cable	.282	28600	257
CFCC^{TM}	.0710	20541	312

Note: HS represents high strength

For the preliminary sizing of the PATRAN models 17-4PH, high strength steel cable and CFCC^{TM} were evaluated. The design that benefitted most, with respect to weight, was the cable dome with a 72% weight savings. The bi-directional and 4-way double layer tensegrity grids only showed a 15.5% and 12.7% weight savings.

Perceptions

All of those authors and researchers who have been referenced in this paper have opinions with respect to tensegrity. For example, Burkhardt lists the four issues, concerns, and reasons why tensegrity has not found its way into mainstream design. The first is strut interference, the second is the poor response of the structure under load, the third is fabrication complexity, and the fourth is inadequate design tools [20]. Snelson comments on the usefulness of tensegrity, “the unfortunate fact of tensegrity is not and never was functional except for the function in my sculptures or permitting viewers to admire the nature of pure structure. . . . the forces in the system need to be so huge that the structure becomes inefficient for supporting any external loads” [18].

Mike Schlaich, designer of the Tower at Rostock, notes that “due to their inherent flexibility and irregularity of the geometry, it is doubtful that also in the future such structures will be much more than impressive sculptures” [20]. The “only practical application has been the so-called ‘cable domes’” [20]. Schlaich also notes that “the potential of tensegrity for roof structures, however, is substantial . . . the increased costs for additional design and fabrication efforts can be compensated by savings in material and weight” [20]. When asked again if he believed his 2004 statements still stand with technological achievements in materials and attachment systems, Schlaich noted that “towers and supports, I think, are generally too flexible to carry relevant loads.” (see Appendix A.1)

Hanaor summarizes the majority opinion by noting that a

> . . . lack of self criticism is a natural human frailty and particularly among engineers and scientists who tend to fall in love with their ideas. It takes courage to admit that a topic you have devoted a large part of your career to research has limited application. Tensegrity is a wonderful topic to research in view of the geometrical complexity and richness of configurations, but its practical application will always be limited to special cases such as space applications and applications of special visual effects (for which there is a price to pay). But the hell with practical application! Just have fun! (see Appendix A.1)

Knowledge Base

For the amount of time invested in this topic, it can be said with certainty that a designer without a thorough knowledge of tensegrity must spend an inordinate amount of time gathering information and then determining fact from fiction. It is even difficult to determine what the most complete definition of tensegrity is. What is lacking is a single source that will serve as a "tensegrity mechanics handbook." This can easily be attributed to the fact that this is, indeed, a blossoming field. At this time I believe that no one publication does an absolutely thorough job, but can say that "Tensegrity Structures and their Application to Architecture" written by Valentín Gómez Jáuregu was extremely helpful in the research conducted for this thesis.

From a practical perspective I would recommend against the use of tensegrity for low budget and short schedule projects. Likewise, for a designer familiar with tensegrity the number of design steps is somewhat more complex than that of conventional structure. For example, the following are typical tensegrity design steps or problems to solve, defined by Motro as "form-finding problems; self stress feasibility, compatibility between self-stress and component stiffness, identification of mechanism, stabilisation of mechanisms, sizing of components, mechanical behaviour under external actions, and sensitivity to imperfections . . . " [26]. However, Motro notes that the addition of prestress effectively reduces the design steps to finding self-stress coefficient values, solving the linear homogeneous system of equilibrium equations and identification of the form required with iteration still required [26].

CHAPTER 3

CABLE DOMES

Configuration Overview

The cable dome structure was modeled in PATRAN, the preliminary geometry is shown in Figure 24, Figure 25, Figure 26 and Figure 27. The structure is composed of seventy two outboard cables, eighty eight inboard cables and ninety two inboard cables; one inboard and outboard hoop cable; twenty two outboard struts and twenty five inboard struts. As with all non-contiguous, k=1, tensegrity structures the struts do not contact one another.

FEM

The PATRAN models types are shown in Figure 27. The cables were modeled using MATD071 nonlinear cable (discrete beam) that is preloaded and then analyzed using SOL700. Struts were modeled as PROD elements and the shelf is a tet10 solid with a load applied at the center of gravity through a Patran MPC (RBE2).

Boundary conditions, shown in Figure 27, specified no translation at the outboard primary structure attach points (represented as ‘123’). For preliminary runs the center nodes for the cables and struts were restricted from moving in the X direction (represented as ‘1’). After the preliminary results (loads and displacement) were confirmed the center

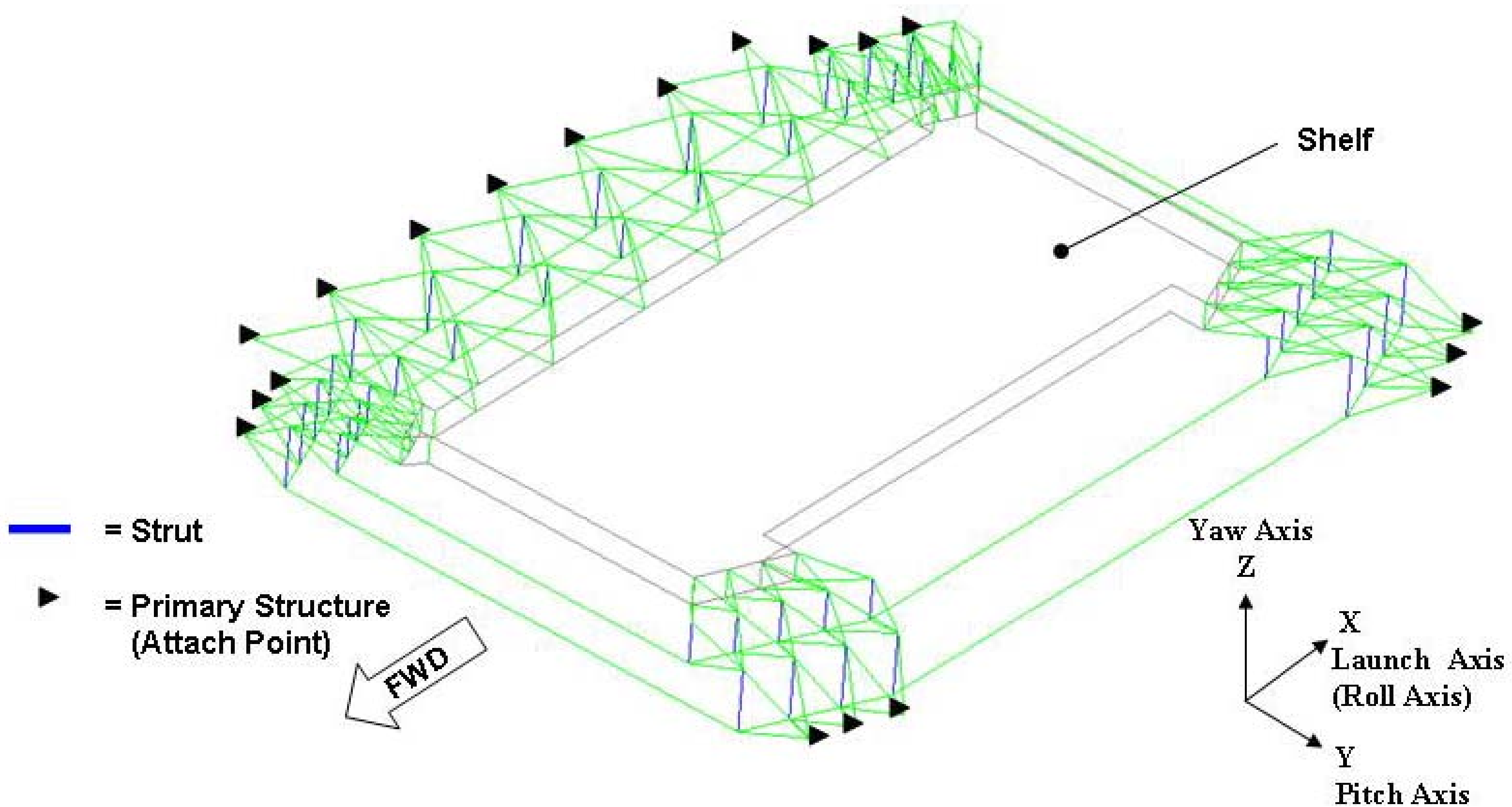


FIGURE 24. Cable dome tensegrity structure (k=1) isometric view (Biele, F.).

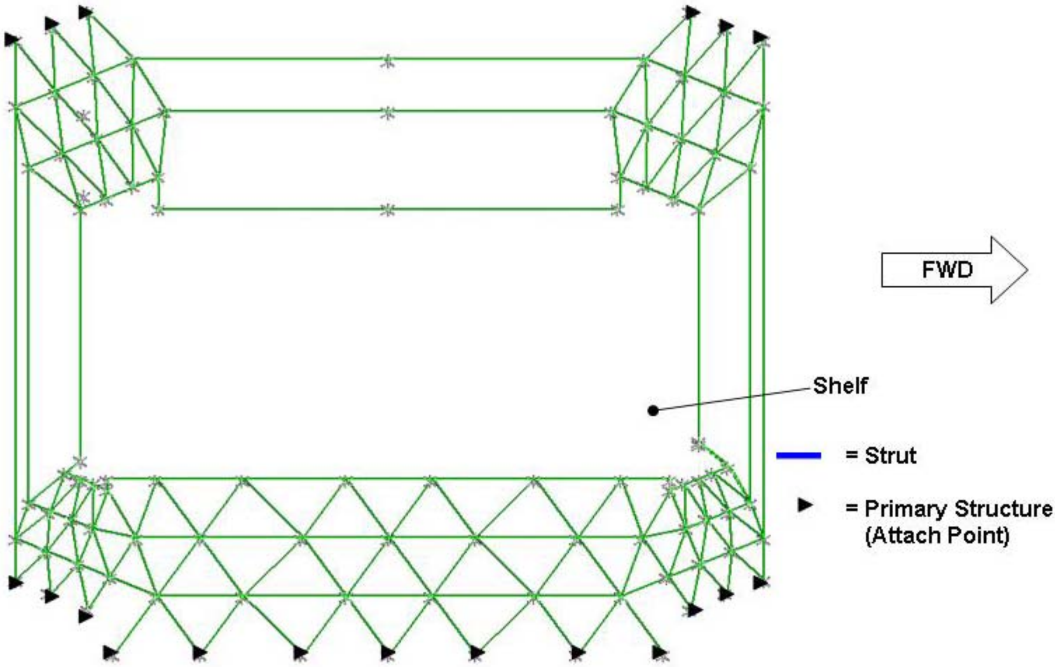


FIGURE 25. Cable dome tensegrity structure top view (Biele, F.).

FIGURE 26. Cable dome tensegrity structure side view (Biele, F.).

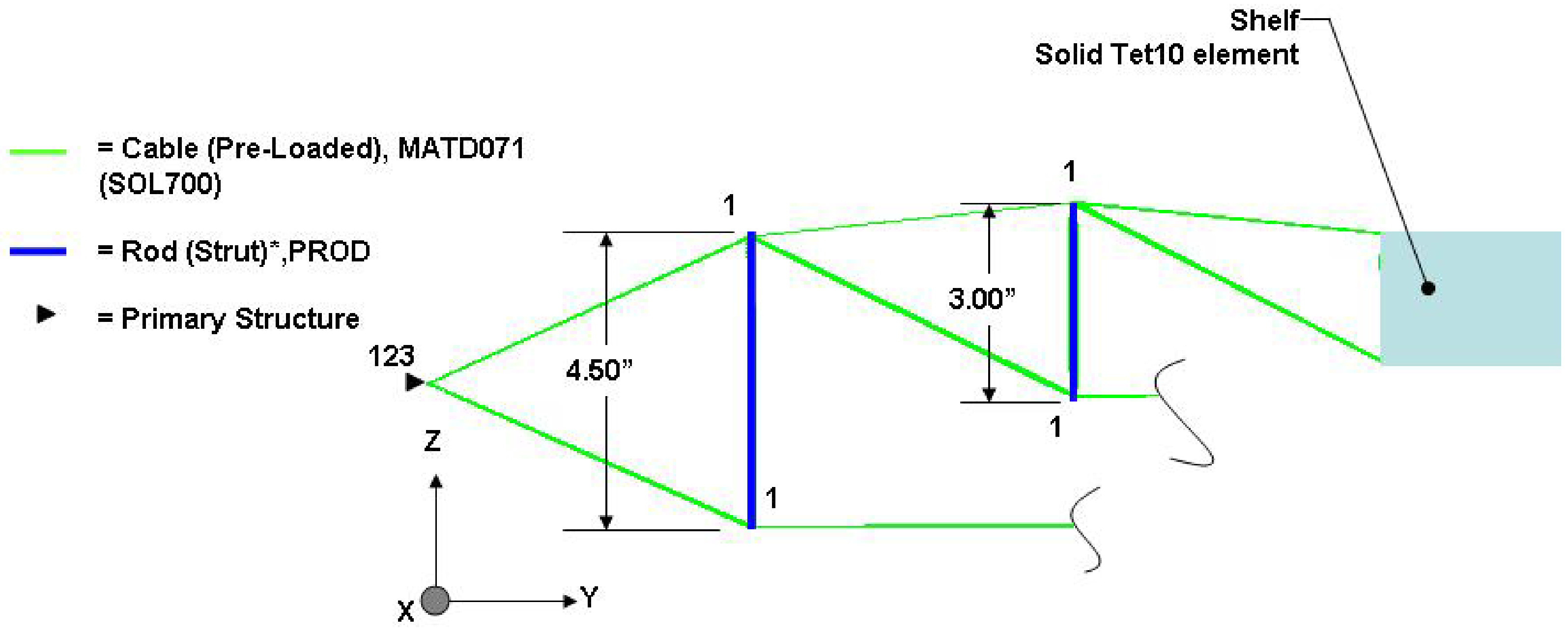


FIGURE 27. Cable dome tensegrity structure view A (Biele, F.).

node X translation restriction was lifted and element forces were found to determine the margin of safety (see Results).

Analysis

Cable dome preliminary cable and strut dimensions are shown in Table 11. This data was used as a starting point for each PATRAN model.

Preliminary Cable Sizing

The cross sectional areas of the cables are derived from the cross sectional area requirements for the baseline design and the tensile strength of the 6AL-4V titanium.

.Baseline Cross Sectional Area = $1.154\text{in}^2 \times 138\text{ksi} = 159{,}291\text{lb}$

Area of Cables = $1.154\text{in}^2 * 138\text{ksi}$ / (Tensile Strength of Cable)

Material. Different cables materials were analyzed. Carbon fiber composite cables (CFCC, see Chapter 1) clearly are the most advantageous with respect to weight and overall strength and therefore were used in the final PATRAN model analyzed.

Preload. Cables were preloaded (prestressed) to 45% of their tensile strength to account for relaxation (see Cable Dome Pre-Load/Prestress). The NATRAN load cases used both self and pre-stress (separately) to determine the optimal loading for the cables.

Preliminary Strut Sizing

The struts were sized by utilizing the a " . . . rigidity ratio (EAstruts/EAcables) close to 10 . . . ". [26]

$$A_{struts}=10 \times E_{cable} \times A_{cable}/E_{strut}$$

Overall length and strut diameter shown are for a uniform cross section. The total weight of the preliminary cable dome support system analyzed is 1.1641 lbs.

TABLE 11. Cable Dome Preliminary Sizing

Description	Strut Dash Number	Cable Dash Number	QTY	Material	Diameter (inch)	Pin to Pin Length (inches)	Wall Thickness (inches)	Sectional Area (in^2)	Height of Uniform Diameter (inches)	I	EI/A	EA	Volume (in^3)	Individual Weight (lbs)	Total Weight (lbs)	EAstrut / EAcable (s/b 10)	Ideal Solid Diameter (inch)	Preload (1/10 tensile strength) (psi)	Preload (45% tensile strength) (psi)
CABLES:																			
Primary Tension		-1	72	CFCC	0.0629	5.39	N/A	3.11E-03	5.39	7.70E-07	5.08E+03	6.39E+04	0.017	0.0012	0.0857	10	0.0629	31200	140400
Middle Tension		-2	88	CFCC	0.0515	5.09	N/A	2.08E-03	5.09	3.45E-07	3.40E+03	4.28E+04	0.011	0.0008	0.0662	N/A	0.0575	31200	140400
Inboard Tension		-3	92	CFCC	0.0557	5.31	N/A	2.43E-03	5.31	4.71E-07	3.98E+03	5.00E+04	0.013	0.0009	0.0844	9	0.0515	31200	140400
Outboard Hoop		-4	1	CFCC	0.0129	204.40	N/A	1.30E-04	204.40	1.35E-09	2.13E+02	2.67E+03	0.027	0.0019	0.0019	N/A	N/A	31200	140400
Inboard Hoop		-5	1	CFCC	0.0129	182.10	N/A	1.30E-04	182.10	1.35E-09	2.13E+02	2.67E+03	0.024	0.0017	0.0017	N/A	N/A	31200	140400
STRUTS																			
Outboard	-1		22	6Al-4V	0.7500	4.50	0.0168	3.87E-02	4.50	2.60E-03	1.11E+06	6.39E+05	0.174	0.0279	0.6133	N/A	0.2220	N/A	N/A
Inboard	-2		25	6Al-4V	0.7500	3.00	0.0112	2.59E-02	3.00	1.77E-03	1.13E+06	4.28E+05	0.078	0.0124	0.3110	N/A	0.1817	N/A	N/A
														Total	1.1641				

Cost

The cable dome design hours are approximated by first determining the hardware count. The total number of struts and cables is shown in Table 11. The struts and cables are installed by an installation drawing that contains the cables, struts, and any attach hardware to primary structure that is required. The hours required to design each detailed piece of hardware are approximated as one hundred hours per detailed drawing, twenty hours per page per installation drawing, and layout drawings are estimated to require 200 hours. A complexity factor of 1.25 was added to the layout to accommodate added time required to determine the optimum layout and avoid potential interferences.

Additional hours include planning and scheduling, as well as design engineering support hours for manufacturing. Each article of released engineering (detailed and installation drawings) require half of an hour per document to maintain and track and two hours per week to update and track the total list. A total of 5% of all hardware manufactured is expected to not conform to drawing requirements and will require four hours to disposition. In addition 3% of the parts will require drawing clarification and will require four hours to disposition.

An itemized list of design hours for the cable dome is shown in Table 12. The total design hours are 1,499 which will require approximately five months to complete (2 persons working full time). The hours may be converted into a total cost by multiplying by $250 an hour.

Material cost was approximated to be thirty dollars per inch cubed for titanium tubing. The total volume of titanium used in the cable dome design is shown in Table 11.

Utilizing the total strut volume we can approximate the material cost as $173. The material cost for CFRP cables is approximated at sixty dollars per inch cubed (approximately two and a half times the cost of high strength steel cable). Therefore the cable material cost is $206. The total cost of both engineering design and material is approximately $375,000.

TABLE 12. Cable Dome Engineering Design Hours

Product	Quantity	Hours
Layout	1 Drawing	250
Strut	2 Drawings	200
Cables	3 Drawings	300
Attach Brackets	3 Drawings (13 Common Brackets)	300
Installation	1 Drawing (15 drawings)	300
Maintain and Track	10 Drawings	5
Update Schedule	20 Weeks	40
Non-Conformance	15 Parts (5% of 312 parts)	64
Drawing Clarification	10 Parts (3% of 312 parts)	40
Total		1499

Note: Design hours are rounded up to the nearest whole hour. Total part quantities are rounded to the nearest whole part for tracking and disposition purposes.

Results

Loads for each element, obtained from NASTRAN, are shown in Table 13. NASTRAN load data was used to verify the margin of safety (MS) for tension, compression and bending (local and Euler) and is shown in Table 14. Calculations are shown in Chapter 1 and are typical for all elements analyzed. Also reference Appendix D for a table of calculations for all cable dome design elements.

The deflection of the preliminary cable dome was a staggering 7.71 plus inches. After resizing the cables and struts to the initial NASTRAN load data the resultant deflection was 3.02 inches or less (61% less than the preliminary, see Appendix D). The penalty for the additional stiffness was a resultant final weight of 3.79 lbs. (see Table 15).

TABLE 13. Element Forces, Cable Dome Structure

	Axial Load (lbs.)		
Description	Liftoff (Limit Load)	Crash Landing (Ultimate Load)	Maximum
568:588,911 (Outboard Struts)	-5626	-5225	-7876
589:613 (Inboard Struts)	-6129	-6075	-8580
614:637,686:717 (Resized Cables)	5445	3808	7623
640:683 (Outboard Cables)	2473	2279	3462
720:771 (Middle Cables)	2262	1933	3727
774:818,820:861 (Inboard Cables)	3206	2698	4488
863:910 (Hoop Cables)	5642	5692	7899

NOTE: Maximum loads reflect an added factor of safety=1.4 .

TABLE 14. Margin of Safety (M.S.) Summary

Description	Failure Mode	M.S.
Outboard Strut	Compression	.0042
Inboard Strut	Compression	.0031
Inboard Cable	Tension	.01
Middle Cable	Tension	.01
Outboard Cable	Tension	.01
Outboard Hoop Cable	Tension	.01
Inboard Hoop Cable	Tension	.01

TABLE 15. Cable Dome Final Sizing

Description	Strut Dash Number	QTY	Tube Number	Diameter (inches)	Pin to Pin Length (inches)	Wall Thickness (inches)	Sectional Area (inches^2)	Height of Uniform Diameter (inch)	I	EI/A	EA	Ti Volume (inches^3)	Individual Weight 6AL-4V (lbs)	Total Weight (lbs)	Prelim Axial Compression Load (lbs)	Prelim Axial Tension Load (lbs)
Outboard	-1	22	6Al-4V	7.50E-01	4.50	2.54E-02	5.77E-02	4.50	3.79E-03	1084351	952592	0.260	4.16E-02	0.9145	-7876	0
Inboard	-2	25	6Al-4V	7.50E-01	3.00	2.77E-02	6.28E-02	3.00	4.10E-03	1077668	1036582	0.188	3.02E-02	0.7539	-8581	0
CABLES																
High Load	-1	67	CFCC	1.77E-01	5.73	N/A	2.47E-02	5.73	4.85E-05	40340	506923	0.141	1.00E-02	0.6727		7623
Primary Tension	-1	44	CFCC	1.19E-01	5.39	N/A	1.12E-02	5.39	1.00E-05	18322	230235	0.060	4.29E-03	0.1887		3462
Middle Tension	-2	88	CFCC	1.24E-01	5.09	N/A	1.21E-02	5.09	1.16E-05	19721	247820	0.061	4.36E-03	0.3834		3727
Inboard Tension	-3	52	CFCC	1.36E-01	5.31	N/A	1.45E-02	5.31	1.68E-05	23747	298409	0.077	5.48E-03	0.2848		4488
Hoop	-4	1	CFCC	1.80E-01	328.20	N/A	2.56E-02	328.20	5.20E-05	41797	525232	8.392	5.96E-01	0.5958		7899
													TOTAL	3.7939		
													Total Weight of Cables	1.2641		
													Cable Percentage of Total Weight	33.32%		

CHAPTER 4

DOUBLE LAYER TENSEGRITY GRIDS

Configuration Overview

Bi-Directional Grids

Bi-directional grids are comprised of simplexes or, as Motro notes for its shape, a'V Expander' (See Figure 30). The simplex is composed of two struts converging at one node into a 'V' shape making contacting with the top and bottom plus an opposite 'V' located perpendicular to the first. The addition of a cable between the two 'V's provides a link and "introduces a self stress" state. [26]

The structure was modeled in PATRAN, the preliminary geometry is shown in Figure 28, Figure 29, Figure 31 and Figure 32. The structure is composed of 72 outboard cables, 88 inboard cables and 92 inboard cables; 1 inboard and outboard hoop cable; 22 outboard struts and 25 inboard struts. As with all contiguous, k=4, tensegrity structures the struts do contact one another. (see Table 16).

FEM

The PATRAN element types are shown in Figure 31. The cables were modeled using MATD071 nonlinear cable (discrete beam) that is preloaded and then analyzed using SOL700. The struts were modeled as PROD elements. The shelf was given a density

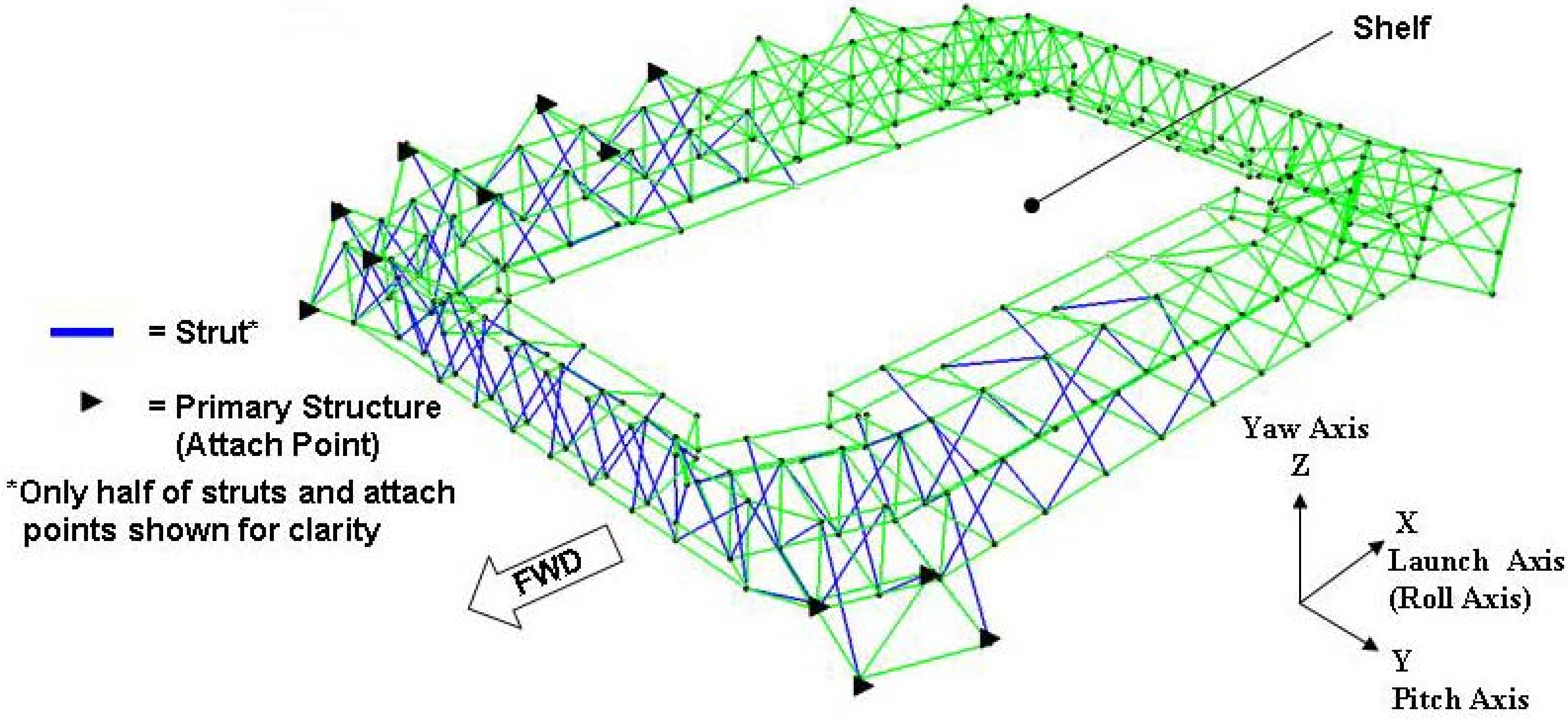


FIGURE 28. Bi-directional double layer tensegrity grid (k>1) isometric view (Biele, F.).

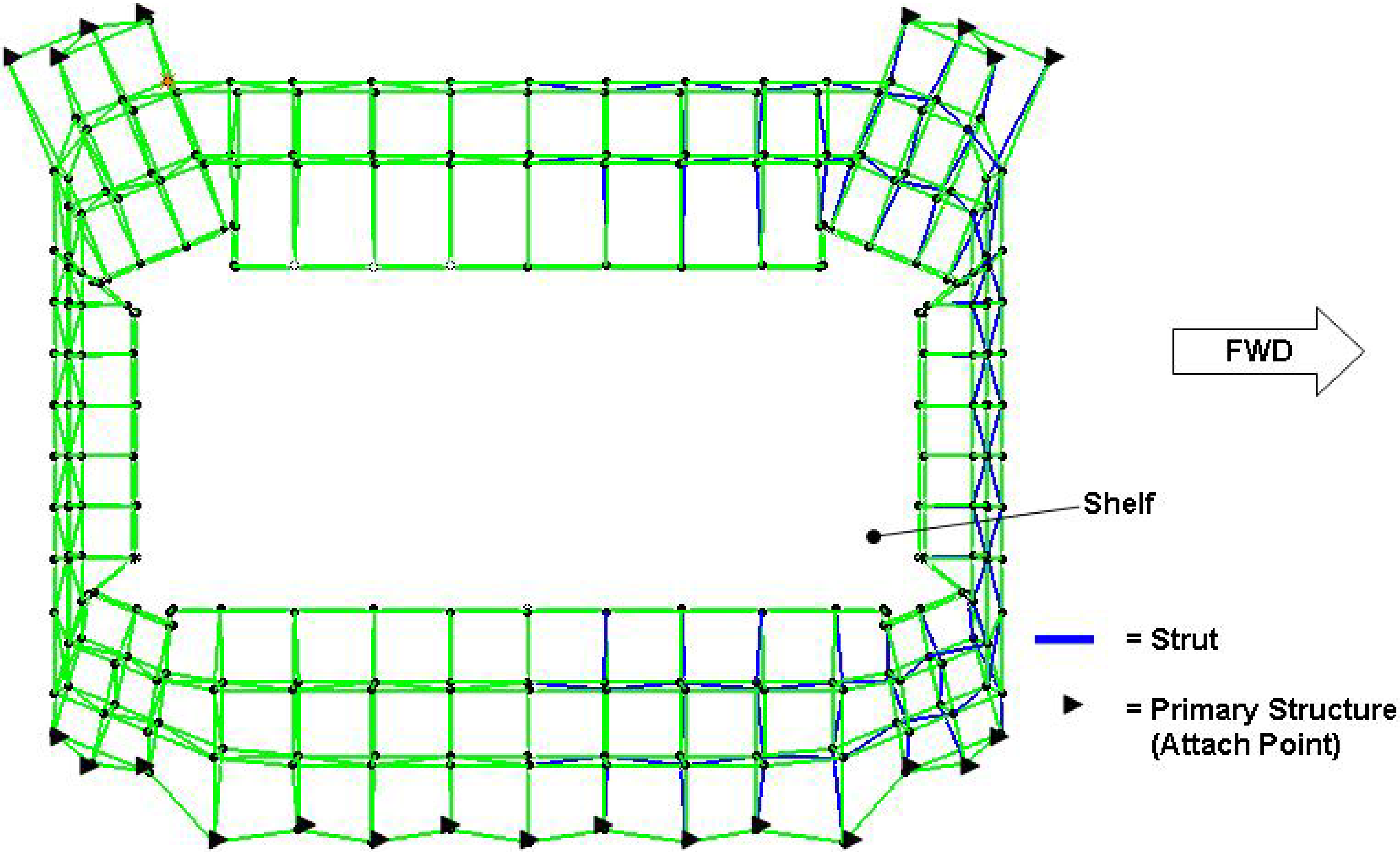


FIGURE 29. Bi-directional double layer tensegrity grid (k>1) top view (Biele, F.).

corresponding to 15 lbs. and was meshed using Tet4 elements. Preliminary boundary conditions were applied as shown in Figure 31 and Figure 36.

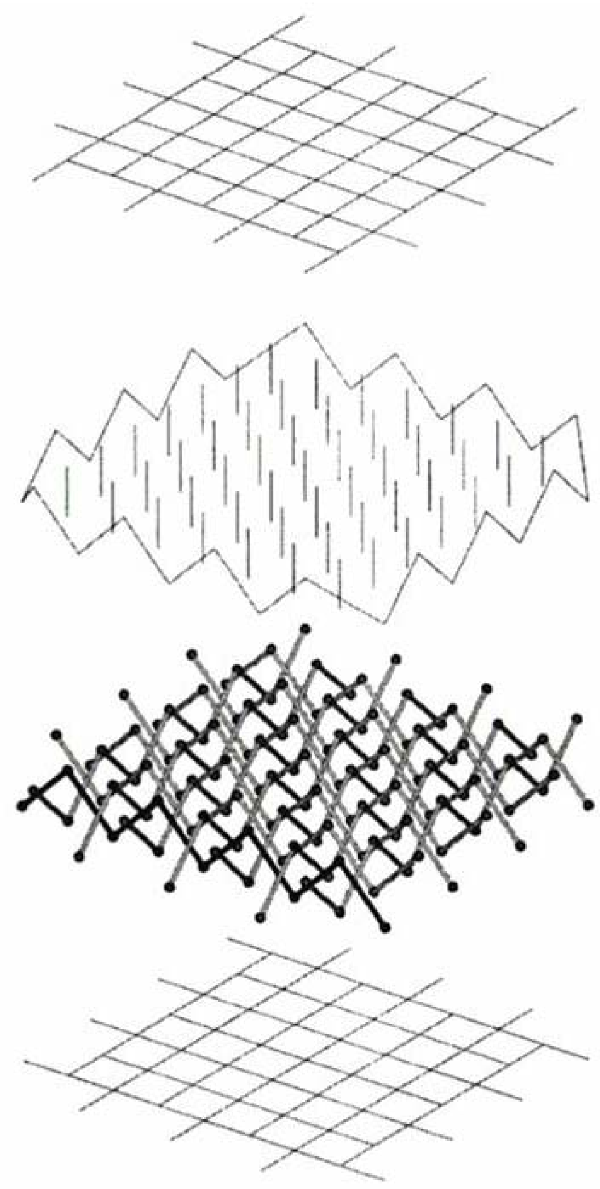


FIGURE 30. Bi-directional double layer tensegrity grid from top to bottom: the upper layer of cables, the bracing of cables, the woven struts, the lower layer of cables (Motro, R., 2003, *Tensegrity: Structural Systems for the Future*, Kogan Page Science, Sterling, VA, Figure 7.10(p.197), Reproduced with permission).

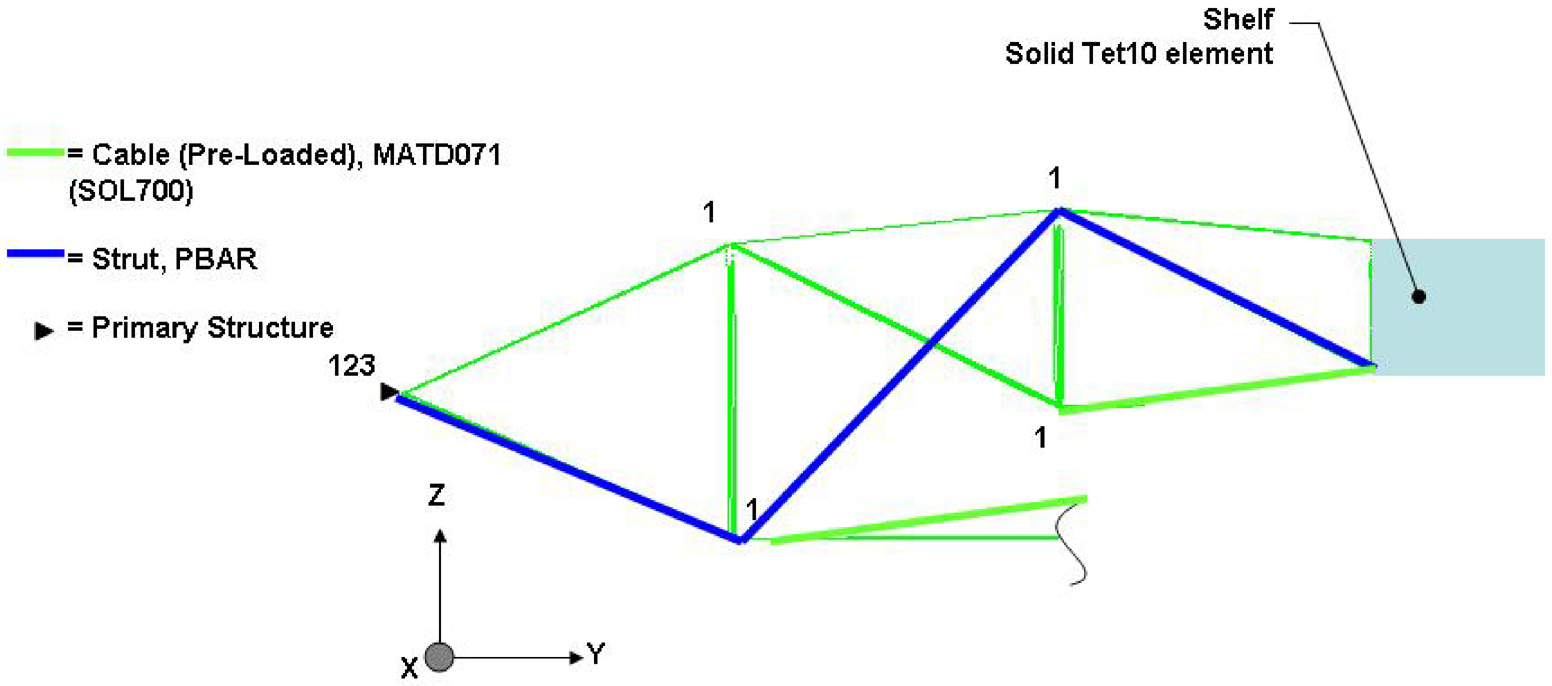


FIGURE 31. Bi-directional double layer tensegrity grid (k>1) cross sectional view (Biele, F.).

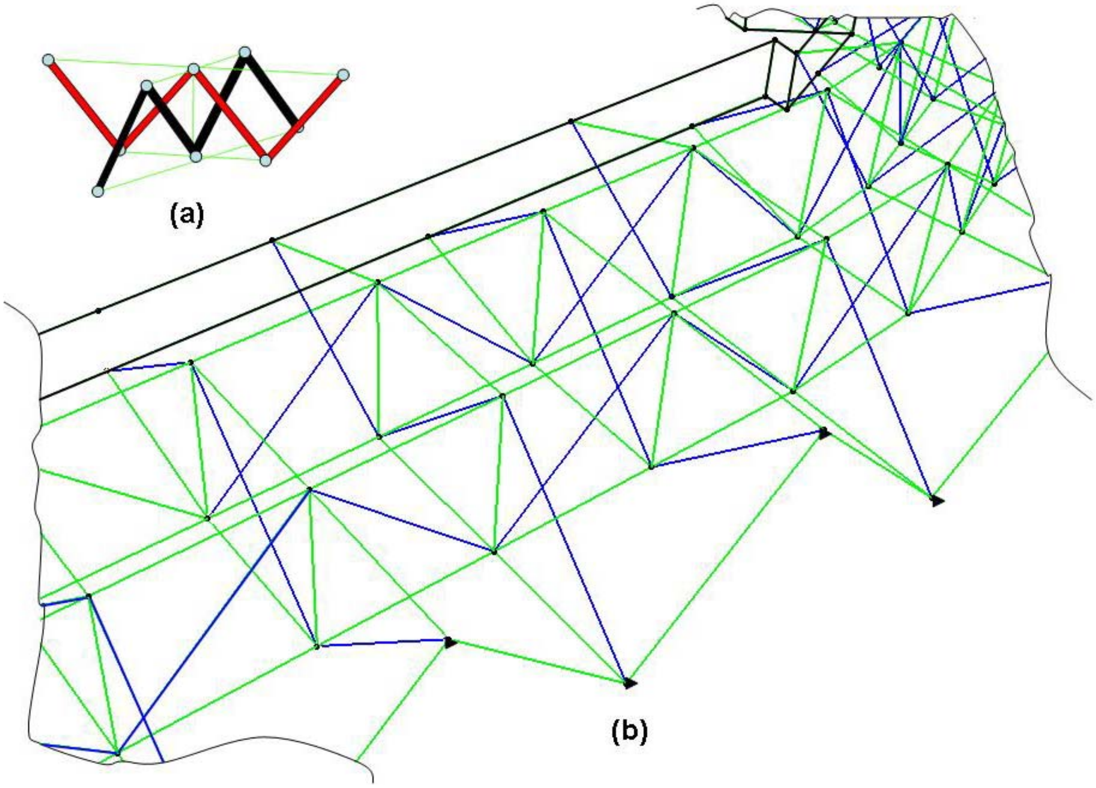


FIGURE 32. Bi-directional double layer tensegrity grid (k>1): (a) simplex; (b) detail view (Biele, F.).

Analysis

General Approach

The typical approach to analysis of tensegrity systems is to first determine the shape, or geometry, of a prestressed structure and use that as a baseline. In the next phase the structural loads are analyzed. As Hanaor notes there are geometrically flexible and geometrically rigid tensegrity structures. Geometrically rigid tensegrity structure can be attained by effectively reducing and eliminating the internal mechanism of the system.[35] More importantly Motro notes, " . . . DLTG's constructed of tensegrity prisms . . . do not involve shape finding, as the prestressed geometry is defined by the prestressed geometry of the individual units" [26]. Therefore, for the purposes of this thesis we will eliminate the shape finding step by preloading the structure to eliminate the internal mechanism.

Bi-Directional Grid

Bi-directional grid preliminary cable and strut dimensions are shown in Table 16. This data was used as a starting point for each PATRAN model. The cross sectional area of the cables was derived from the sectional requirements for the baseline design. In addition different cables materials were analyzed. CRCC Cables (see Chapter 1) clearly are the most advantageous with respect to weight and overall strength and therefore were used in the final PATRAN model analyzed.

Preliminary Cable Sizing

The cross sectional areas of the cables are derived from the cross sectional area requirements for the baseline design and the tensile strength of the 6AL-4V titanium. (CFCC cable sample calculations shown below).

$$\text{Baseline Cross Sectional Area} = 1.154\text{in}^2 \ \text{x}\ 138\text{ksi} = 159{,}291\text{lb}$$

$$\text{Area}_{\text{strut}} = 10 \ \text{x}\ E_{\text{cable}} \ \text{x}\ A_{\text{cable}}/E_{\text{strut}} = 17.333 * A_{\text{cable}}$$

$$\text{Area}_{\text{required}} = 1.154\text{in}^2 * 138\text{ksi} = A_{\text{cable}} \ \text{x}\ 312\ \text{ksi} + A_{\text{strut}} * 138\text{ksi} = 2030\ \text{ksi} * A_{\text{cable}}$$

Material. Different cables materials were analyzed. Carbon fiber composite Cables (CFCC, see Chapter 1) clearly are the most advantageous with respect to weight and overall strength and therefore were used in the final PATRAN model analyzed.

TABLE 16. Bi-Directional Grid Preliminary Sizing

Description	Strut Dash Number	Cable Dash Number	QTY	Material	Diameter (inch)	Pin to Pin Length (inches)	Wall Thickness (inches)	Sectional Area (in^2)	Height of Uniform Diameter (inches)	I	EI/A	EA	Volume (in^3)	Individual Weight (lbs)	Total Weight (lbs)	EAstrut / EAcable (s/b 10)	Ideal Solid Diameter (inch)	Preload (1/10 tensile strength) (psi)	Preload (45% tensile strength) (psi)
CABLES:																			
Primary Tension		-1	23	17-4PH	0.0436	11.74	N/A	1.50E-03	11.74	1.78E-07	2446	30734	1.756E-02	1.25E-03	0.0287	10.00	0.0436	31200	140400
Middle Tension		-2	276	17-4PH	0.0036	4.49	N/A	1.04E-05	4.49	8.59E-12	17	213	4.664E-05	3.31E-06	0.0009	N/A	N/A	31200	140400
Inboard Tension		-3	46	17-4PH	0.0309	6.22	N/A	7.48E-04	6.22	4.45E-08	1223	15367	4.651E-03	3.30E-04	0.0152	9.58	N/A	31200	140400
STRUTS:																			
Y-AXIS Outboard	-1		23	6Al-4V	0.3500	6.91	0.0179	1.86E-02	6.91	2.58E-04	228197	307361	1.288E-01	2.06E-02	0.4739	N/A	N/A	N/A	N/A
Middle	-2		48	6Al-4V	0.1677	7.20	0.0042	2.14E-03	7.20	7.15E-06	55202	35285	1.539E-02	2.46E-03	0.1182	N/A	N/A	N/A	N/A
Inboard	-3		48	6Al-4V	0.3000	4.80	0.0098	8.93E-03	4.80	9.41E-05	173905	147277	4.288E-02	6.86E-03	0.3293	N/A	N/A	N/A	N/A
Hoop (X-axis)	-4		100	6Al-4V	0.0805	5.18	0.0008	2.05E-04	5.18	1.63E-07	13096	3387	1.064E-03	1.70E-04	0.0170	N/A	N/A	N/A	N/A
													Total		0.9832				

Preload. Cables were preloaded (prestressed) to 45% of their tensile strength to account for relaxation (see Cable Pre-Load/Prestress). The NATRAN load cases used both self and pre-stress (separately) to determine the optimal loading for the cables.

Preliminary Strut Sizing

The struts were sized by utilizing the a "... rigidity ratio (EAstruts/EAcables) close to 10 . . . " [26].

$$A_{struts}=10 \times E_{cable} \times A_{cable}/E_{strut}$$

Overall length and strut diameter shown are for a uniform cross section. The total weight of the Bi-Directional DLGT support system analyzed is 0.9832 lbs.

Bi-Directional Cost

The bi-directional design hours are approximated by first determining the hardware count. The total number of struts and cables is shown in Table 16. The struts and cables are installed by an installation drawing that contains the cables, struts, and any attach hardware to primary structure that is required. The hours required to design each detailed piece of hardware are approximated as one hundred hours per detailed drawing, twenty hours per page per installation drawing, and layout drawings are estimated to require 200 hours. A complexity factor of 1.40 was added to the layout to accommodate added time required to determine the optimum layout and avoid potential interferences.

Additional hours include planning and scheduling, as well as design engineering support hours for manufacturing. Each article of released engineering (detailed and installation drawings) require half of an hour per document to maintain and track and two hours per week to update and track the total list. A total of 5% of all hardware manufactured is expected to not conform to drawing requirements and will require four hours to disposition. In addition 3% of the parts will require drawing clarification and will require four hours to disposition.

An itemized list of design hours for the bi-directional grid is shown in Table 17. The total design hours are 1,882 which will require approximately six months to complete (2 persons working full time). The hours may be converted into a total cost by multiplying by $250 an hour.

Material cost was approximated to be thirty dollars per cubic inch for titanium tubing. The total volume of titanium used in the cable dome design is shown in Table 16. Utilizing the total strut volume we can approximate the material cost as $176. The material cost for CFRP cables is approximated at sixty dollars per inch cubed (approximately two

TABLE 17. Bi-Directional Engineering Design Hours

Product	Quantity	Hours
Layout	1 Drawing	280
Strut	4 Drawings	400
Cables	3 Drawings	300
Attach Brackets	3 Drawings (14 Common Brackets)	300
Installation	1 Drawing (18 pages)	360
Maintain and Track	12 Drawings	6
Update Schedule	24 Weeks	48
Non-Conformance	29 Parts (5% of 577 parts)	116
Drawing Clarification	18 Parts (3% of 577 parts)	72
Total		1882

Note: Design hours are rounded up to the nearest whole hour. Total part quantities are rounded to the nearest whole part for tracking and disposition purposes.

and a half times the cost of high strength steel cable). Therefore the cable material cost is $256. The total cost of both engineering design and material is approximately $471,000.

Bi-Directional Results

Loads for each element, obtained from NASTRAN, are shown in Table 18. NASTRAN load data was used to verify the margin of safety (MS) for tension, compression and bending (local and Euler) and is shown in Table 20. Calculations are shown in Chapter 1 and are typical for all elements analyzed. Also reference Appendix E for a table of calculations for all bi-directional design elements.

TABLE 18. Element Forces, Bi-Directional Structure

STRUT ELEMENT FORCES			
	Axial Load (lbs.)		
Description	Liftoff (Limit Load)	Crash Landing (Ultimate Load)	Maximum
Outboard Struts	-1588	-1304	-2224
Middle Struts	-948	-850	-1327
Inboard Struts	-1248	-961	-1747
Hoop Struts	-612	-865	-865
Outboard Cables	2445	2081	3423
Middle Cables	1883	1200	2636
Inboard Cables	1804	1394	2525

NOTE: Maximum loads reflect an added factor of safety=1.4 .

The deflection of the preliminary bi-directional grid was a 2.37 plus inches. After resizing the cables and struts to the initial NASTRAN load data the resultant deflection was

TABLE 19. Bi-Directional Grid Final Sizing

Description	Strut Dash Number	QTY	Tube Number	Diameter (inches)	Pin to Pin Length (inches)	Wall Thickness (inches)	Sectional Area (inches^2)	Height of Uniform Diameter (inch)	I	EI/A	EA	Ti Volume (inches^3)	Individual Weight 6AL-4V (lbs)	Total Weight (lbs)	Prelim Axial Compress-ion Load (lbs)	Prelim Axial Tension Load (lbs)
STRUTS																
Outboard	-1	23	6Al-4V	8.10E-01	6.91	6.45E-03	1.63E-02	6.91	1.31E-03	1331838	268524	0.113	1.80E-02	0.4140	-2224	0
Middle	-2	48	6Al-4V	9.40E-01	7.20	3.42E-03	1.01E-02	7.20	1.10E-03	1809227	165851	0.072	1.16E-02	0.5556	-1327	0
Inboard	-3	44	6Al-4V	7.50E-01	5.24	4.16E-03	9.75E-03	5.24	6.78E-04	1147355	160863	0.051	8.18E-03	0.3597	-1747	0
Hoop	-4	186	6Al-4V	4.00E-01	2.79	1.04E-02	1.28E-02	2.79	6.74E-05	177394	103436	0.036	5.70E-03	1.0600	-865	0
CABLES																
Primary Tension	-1	23	17-4PH	1.19E-01	11.74	N/A	1.11E-02	11.74	9.77E-06	18113	227610	0.130	9.24E-03	0.2124		3423
Middle Tension	-2	346	17-4PH	1.04E-01	3.58	N/A	8.53E-03	3.58	5.79E-06	13947	175261	0.031	2.17E-03	0.7506		2636
Inboard Tension	-3	45	17-4PH	1.02E-01	6.36	N/A	8.17E-03	6.36	5.32E-06	13362	167911	0.052	3.69E-03	0.1660		2525
													TOTAL	3.5183		
													Total Weight of Cables	1.1290		
													Weight of Cables/Total Weight	32.09%		

0.525 inches or less (78% less than the preliminary, see Appendix E). The penalty for the additional stiffness was a resultant final weight of 3.52 lbs. (see Table 19).

TABLE 20. Margin of Safety (M.S.) Summary

Description	Failure Mode	M.S.
Outboard Strut	Compression	0.003
Middle Strut	Local Buckling	0.002
Inboard Strut	Compression	0.006
Hoop Strut	Compression	0.003
Outboard Cable	Tension	0.01
Middle Cable	Tension	0.01
Inboard Cable	Tension	0.01

4-Way Grids

4-Way grids are modeled like the previously shown bi-directional grid only with 2 additional diagonal elements added to the ‘V expander’ or simplex. The grid is comprised of simplexes or, as Motro notes for its shape, a ‘V Expander’. The simplex is composed of four struts converging at one node , each in a ‘V’ shape, making contacting with the top and bottom plus an opposite 4 strut simplex located perpendicular to the first. (See Figure 35) The addition of a cable between the two four strut simplex provides a link and “introduces a self stress” state. [26]

The structure was modeled in PATRAN and the preliminary geometry is summarized in Figure 33, Figure 34, Figure 36 and Figure 37. The structure is composed of forty nine outboard cables, 256 inboard cables and sixty inboard cables; eighty six inboard and outboard hoop cables; twenty three outboard struts and fifty four middle and thirty eight inboard struts; eighteen outboard diagonal

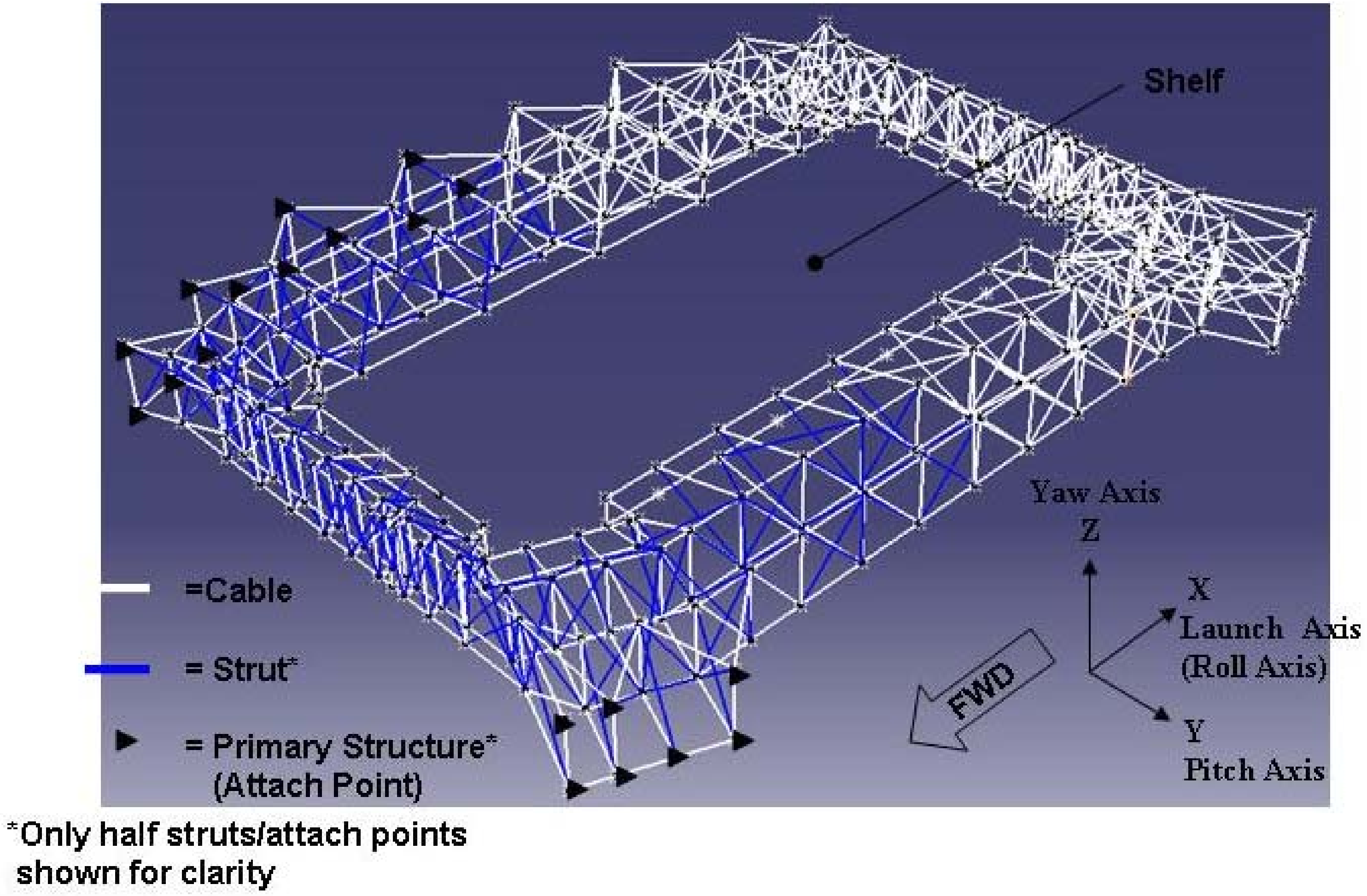


FIGURE 33. 4-Way double layer tensegrity grid isometric view (Biele, F.).

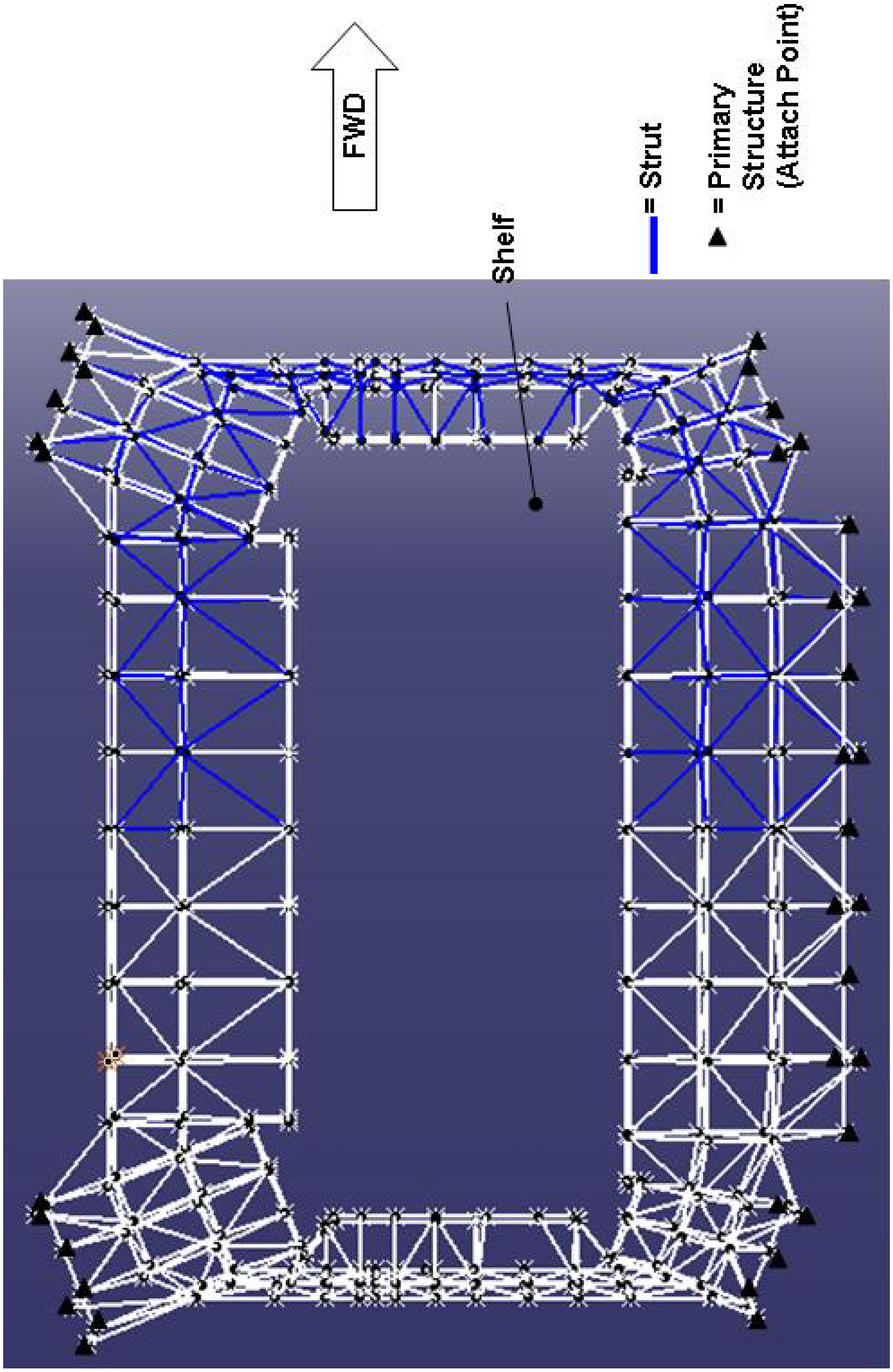


FIGURE 34. 4-Way double layer tensegrity grid (k>1) top (Biele, F.).

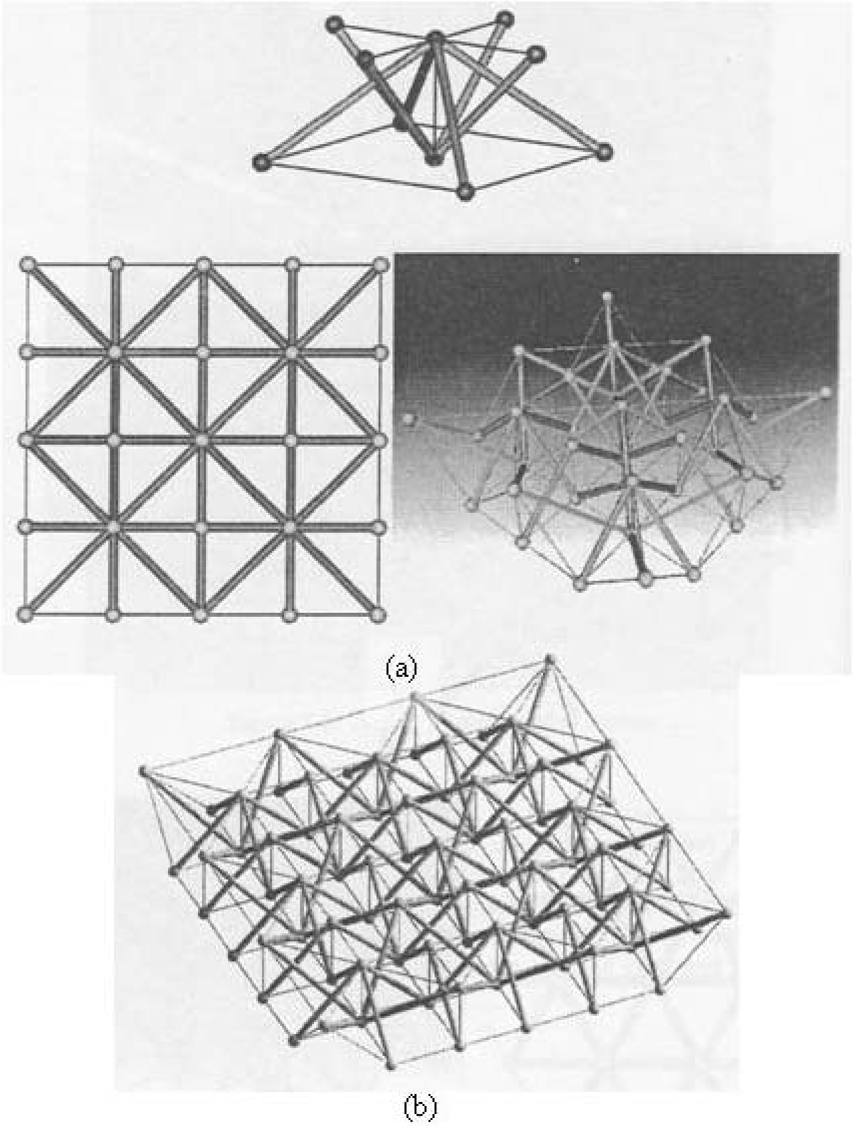


FIGURE 35. 4-Way double layer tensegrity grid (k>1): (a) Motro 4-Way simplex ; (b) complete Motro 4-Way grid (Motro, R., 2003, *Tensegrity: Structural Systems for the Future*, Kogan Page Science, Sterling, VA, Figure 7.17 (p.202), Figure 7.18(p.202) . Reproduced with permission).

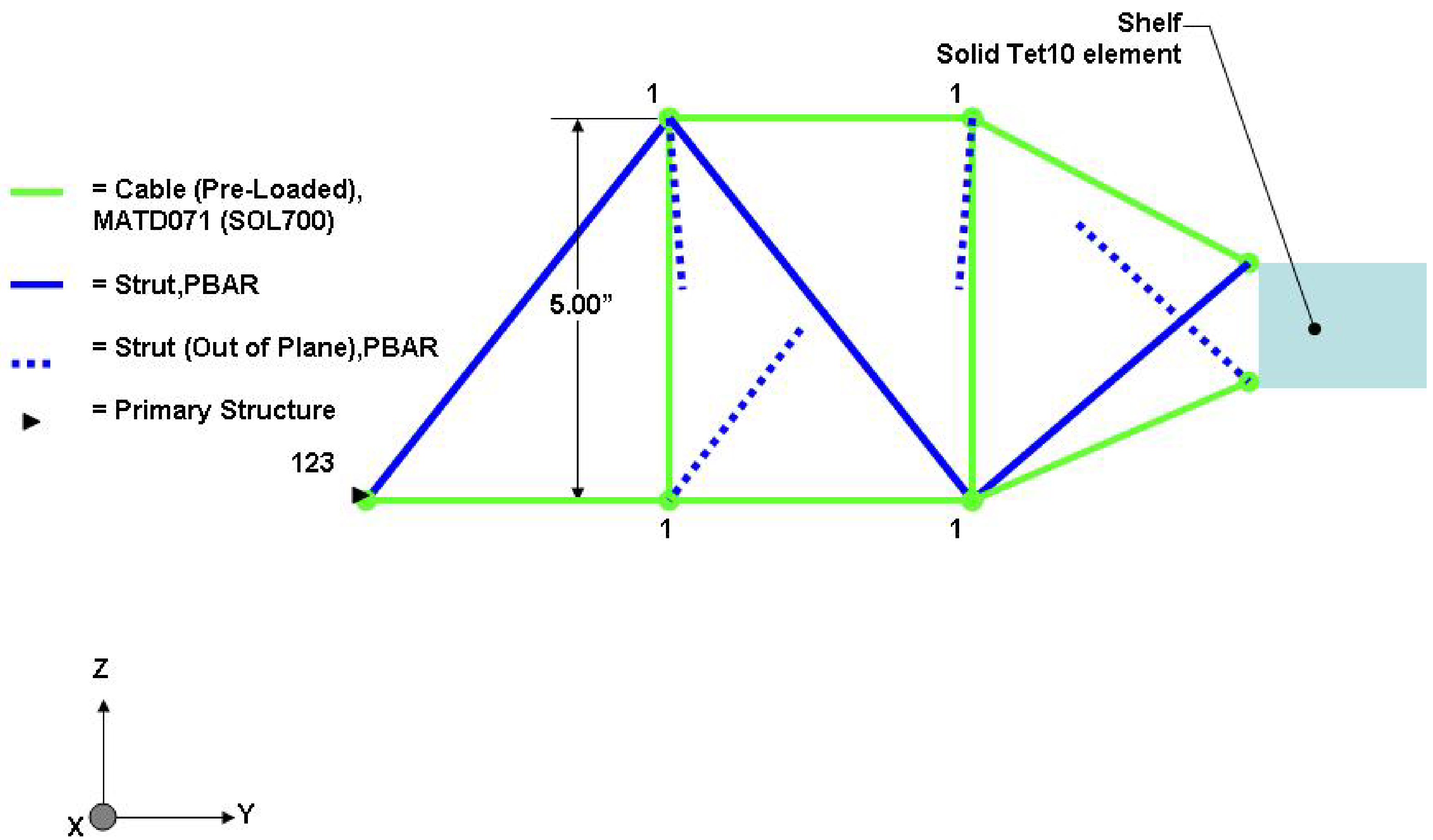


FIGURE 36. 4-Way double layer tensegrity grid side view (Biele, F.).

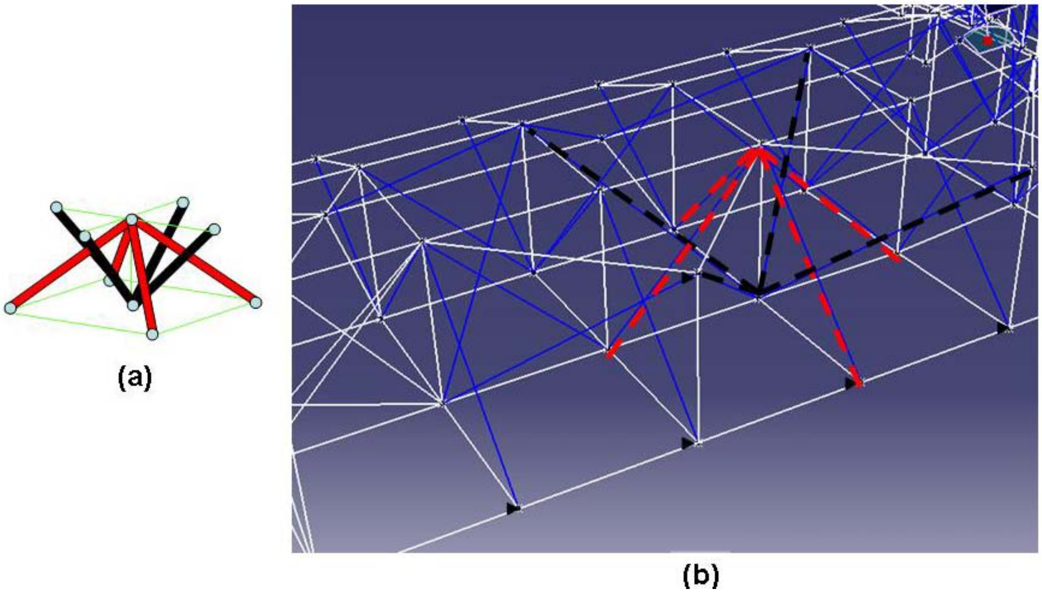


Figure 37. 4-Way Way double layer tensegrity grid (k>1): (a) simplex; (b) detail view (Biele, F.).

struts, forty two middle diagonal and thirty six inboard diagonal struts. As with all contiguous, k=4, tensegrity structures the struts do contact one another. (see Table 19)

The PATRAN element types are shown in Figure 36. The cables were modeled using MATD071 nonlinear cable (discrete beam) that is preloaded and then analyzed using SOL700. Struts were modeled as PROD elements and the shelf is a tet10 solid with a load applied at the center of gravity through an Patran MPC (RBE2).

4-Way Analysis

4-Way Grid

4-Way grid preliminary cable and strut dimensions are shown in Table 21. This data was used as a starting point for each PATRAN model. The cross sectional area of the cables was derived from the sectional requirements for the baseline design. In addition different cables materials were analyzed. CRCC Cables (see Chapter 1) clearly are the most advantageous with respect to weight and overall strength and therefore were used in the final PATRAN model analyzed.

Preliminary Cable Sizing

The cross sectional areas of the cables are derived from the cross sectional area requirements for the baseline design and the tensile strength of the 6AL-4V titanium. (CFCC cable sample calculations shown below)

$$.\text{Baseline Cross Sectional Area} = 1.154\text{in}^2 \ \text{x} \ 138\text{ksi} = 159{,}291\text{lb}$$

$$\text{Area}_{\text{strut}} = 10 \ \text{x} \ E_{\text{cable}} \ \text{x} \ A_{\text{cable}}/E_{\text{strut}} = 17.333 * A_{\text{cable}}$$

$$\text{Area}_{\text{ required}} = 1.154\text{in}^2 * 138\text{ksi} = A_{\text{cable}} \ \text{x} \ 312 \ \text{ksi} + A_{\text{strut}} * 138 \ \text{ksi} = 2030 \ \text{ksi} * A_{\text{cable}}$$

Material. Different cables materials were analyzed. Carbon fiber composite Cables (CFCC, see Chapter 1) clearly are the most advantageous with respect to weight and overall strength and therefore were used in the final PATRAN model analyzed.

Preload. Cables were preloaded (prestressed) to 45% of their tensile strength to account for relaxation (see Cable Pre-Load/Prestress). The NATRAN load cases used both self and pre-stress (separately) to determine the optimal loading for the cables.

Preliminary Strut Sizing

The struts were sized by utilizing the a " . . . rigidity ratio (EAstruts/EAcables) close to 10 . . . " [26].

$$A_{\text{struts}} = 10 \ \text{x} \ E_{\text{cable}} \ \text{x} \ A_{\text{cable}}/E_{\text{strut}}$$

Overall length and strut diameter shown are for a uniform cross section. The total weight of the 4-Way DLGT support system analyzed is 1.1054 lbs. (see Table 21).

4-Way Cost

The 4-way grid design hours are approximated by first determining the hardware count. The total number of struts and cables is shown in Table 21. The struts and cables are installed by an installation drawing that contains the cables, struts, and any attach hardware to primary structure that is required. The hours required to design each detailed piece of hardware are approximated as one hundred hours per detailed drawing, twenty hours per page per installation drawing, and layout drawings are estimated to require two hundred hours. A complexity factor of 1.6 was added to the layout to accommodate added time required to determine the optimum layout and avoid potential interferences.

Additional hours include planning and scheduling, as well as design engineering support hours for manufacturing. Each article of released engineering (detailed and installation drawings) require half of an hour per document to maintain and track and two hours per week to update and track the total list. A total of five percent of all hardware manufactured is expected to not conform to drawing requirements and will require four hours to disposition. In addition three percent of the parts will require drawing clarification and will require four hours to disposition.

An itemized list of design hours for the 4-way grid are shown in Table 22. The total design hours are two thousand four hundred ninety two which will require approximately eight months to complete (2 persons working full time). The hours may be converted into a total cost by multiplying by two hundred and fifty dollars an hour.

Material cost was approximated to be thirty dollars per inch cubed for titanium tubing. The total volume of titanium used in the 4 way grid design is shown in Table 21.

TABLE 21. 4-Way grid preliminary sizing.

Description	Strut Dash Number	Cable Dash Number	QTY	Material	Diameter (inch)	Pin to Pin Length (inches)	Wall Thickness (inches)	Sectional Area (in^2)	Height of Uniform Diameter (inches)	I	EI/A	EA	Volume (in^3)	Individual Weight (lbs)	Total Weight (lbs)	EAstrut/EAcable (s/b 10)	Ideal Solid Diameter (inch)	Preload (1/10 tensile strength) (psi)	Preload (45% tensile strength) (psi)
CABLES:																			
Primary Tension		-1	49	CFCC	2.99E-02	5.51	N/A	7.02E-04	5.51	3.93E-08	1148	14427	3.87E-03	2.75E-04	0.0135	9	0.02451	31200	140400
Middle Tension		-2	256	CFCC	5.72E-03	4.84	N/A	2.57E-05	4.84	5.27E-11	42	529	1.25E-04	8.84E-06	0.0023	N/A	N/A	31200	140400
Inboard Tension		-3	60	CFCC	2.70E-02	4.77	N/A	5.74E-04	4.77	2.62E-08	938	11782	2.73E-03	1.94E-04	0.0116	8	N/A	31200	140400
STRUTS:																			
Y-AXIS																			
Outboard	-1		23	6Al-4V	3.50E-01	6.91	7.60E-03	8.16E-03	6.91	1.20E-04	241918	134946	5.65E-02	9.05E-03	0.2081	N/A	N/A	N/A	N/A
Middle	-2		54	6Al-4V	1.49E-01	6.40	1.60E-03	7.43E-04	6.40	2.02E-06	44860	12253	4.75E-03	7.60E-04	0.0410	N/A	N/A	N/A	N/A
Inboard	-3		38	6Al-4V	3.00E-01	6.07	5.94E-03	5.49E-03	6.07	5.93E-05	178423	90508	3.33E-02	5.33E-03	0.2024	N/A	N/A	N/A	N/A
Hoop (X-axis)	-4		86	6Al-4V	9.36E-02	6.02	3.18E-04	9.32E-05	6.02	1.01E-07	17949	1537	5.61E-04	8.98E-05	0.0077	N/A	N/A	N/A	N/A
DIAGONAL																			
Outboard	-5		18	6Al-4V	3.50E-01	7.85	1.26E-02	1.34E-02	7.85	1.90E-04	235123	220328	1.05E-01	1.68E-02	0.3018	N/A	N/A	N/A	N/A
Middle	-6		42	6Al-4V	1.50E-01	7.06	2.65E-03	1.23E-03	7.06	3.33E-06	44796	20234	8.66E-03	1.38E-03	0.0582	N/A	N/A	N/A	N/A
Inboard	-7		36	6Al-4V	3.00E-01	7.35	6.63E-03	6.11E-03	7.35	6.58E-05	177600	100843	4.49E-02	7.19E-03	0.2588	N/A	N/A	N/A	N/A
													Total		1.1054				

Utilizing the total titanium we can approximate the total material cost as two hundred and two dollars. The material cost for CFRP cables is approximated at sixty dollars per inch cubed (approximately two and a half times the cost of high strength steel cable). Therefore the cable material cost is twenty three dollars. The total cost of both engineering design and material is approximately three six hundred and twenty three thousand dollars.

TABLE 22. 4 Way Engineering Design Hours

Product	Quantity	Hours
Layout	1 Drawing	320
Strut	7 Drawings	700
Cables	3 Drawings	300
Attach Brackets	4 Drawings (17 Common Brackets)	400
Installation	1 Drawing (24 pages)	480
Maintain and Track	16 Drawings	8
Update Schedule	32 Weeks	56
Non-Conformance	34 Parts (5% of 679 parts)	136
Drawing Clarification	21 Parts (3% of 679 parts)	84
Total		2492

Note: Design hours are rounded up to the nearest whole hour. Total part quantities are rounded to the nearest whole part for tracking and disposition purposes.

4-Way Results

Loads for each element, obtained from NASTRAN, are shown in Table 23. NASTRAN load data was used to verify the margin of safety (MS) for tension, compression and bending (local and Euler) and is shown in Table 24. Calculations are shown in Chapter 1 and are typical for all elements analyzed. Also reference Appendix F for a table of calculations for all cable dome design elements.

TABLE 23. Element Forces, 4-Way Structure

	Axial Load (lbs.)		
Description	Liftoff (Limit Load)	Crash Landing (Ultimate Load)	Maximum
Outboard Struts	-857	-614	-1200
Middle Struts	-462	-236	-646
Inboard Struts	-842	-593	-1179
Hoop Struts	-420	-420	-588
Diagonal Struts			
Outboard Struts	-624	-638	-873
Middle Struts	-561	-357	-785
Inboard Struts	-564	-522	-790
Cables			
Outboard Cables	920	950	1288
Middle Cables	656	678	918
Inboard Cables	659	653	923
High Load Cables	1162	950	1627

NOTE: Maximum loads reflect an added factor of safety=1.4 .

TABLE 24. Margin of Safety (M.S.) Summary

Description	Failure Mode	M.S.
Outboard Strut	Local Buckling	0.006
Middle Strut	Local Buckling	0.009
Inboard Strut	Compression	0.01
Hoop Strut	Compression	0.01
Diagonal Struts		
Outboard Strut	Local Buckling	0.0002
Middle Strut	Local Buckling	0.007
Inboard Strut	Local Buckling	0.001
Cables		
Outboard Cable	Tension	0.01
Middle Cable	Tension	0.01
Inboard Cable	Tension	0.01

TABLE 25. 4-Way grid final sizing.

Description	Strut Dash Number	QTY	Tube Number	Diameter (inches)	Pin to Pin Length (inches)	Wall Thickness (inches)	Sectional Area (inches^2)	Height of Uniform Diameter (inch)	I	EI/A	EA	Ti Volume (inches^3)	Individual Weight 6AL-4V (lbs)	Total Weight (lbs)	Prelim Axial Compression Load (lbs)	Prelim Axial Tension Load (lbs)
Struts																
Outboard	-1	23	6Al-4V	9.70E-01	6.91	3.04E-03	9.24E-03	6.91	1.08E-03	1928475	152450	0.064	1.02E-02	2.35E-01	-1200	0
Middle	-2	48	6Al-4V	6.90E-01	7.20	3.48E-03	7.50E-03	7.20	4.42E-04	972113	123695	0.054	8.63E-03	4.14E-01	-646	0
Inboard	-3	43	6Al-4V	6.50E-01	5.36	4.30E-03	8.72E-03	5.36	4.54E-04	859963	143799	0.047	7.48E-03	3.22E-01	-1179	0
Hoop	-4	186	6Al-4V	3.00E-01	2.79	3.68E-03	4.35E-03	2.79	3.76E-05	142626	71708	0.012	1.94E-03	3.60E-01	-588	0
Outboard Diagonal	-5	22	6Al-4V	7.90E-01	6.42	3.05E-03	7.53E-03	6.42	5.83E-04	1277320	124236	0.048	7.74E-03	1.70E-01	-873	0
Middle Diagonal	-6	52	6Al-4V	7.70E-01	5.70	2.60E-03	6.43E-03	5.70	4.61E-04	1183782	106026	0.037	5.86E-03	3.05E-01	-785	0
Inboard Diagonal	-7	52	6Al-4V	6.25E-01	5.09	3.02E-03	5.90E-03	5.09	2.85E-04	797921	97296	0.030	4.80E-03	2.50E-01	-790	0
Outboard Cables	-1	49	CFCC	7.29E-02	5.51	N/A	4.17E-03	5.51	1.38E-06	6818	85677	0.023	1.63E-03	8.00E-02		1288
Cables	-2	414	CFCC	6.15E-02	2.99	N/A	2.97E-03	2.99	7.03E-07	4860	61073	0.009	6.32E-04	2.62E-01		918
Inboard Cables	-3	64	CFCC	6.17E-02	4.47	N/A	2.99E-03	4.47	7.11E-07	4885	61385	0.013	9.48E-04	6.07E-02		923
New High Load Cable	-3	64	CFCC	8.19E-02	4.47	N/A	5.27E-03	4.47	2.21E-06	8611	108207	0.024	1.67E-03	1.07E-01		1627
													TOTAL	2.57E+00		
													Total Weight of Cables	4.29E-01		
													Weight of Cables/Total Weight	1.67E-01		

The deflection of the preliminary cable dome was 2.26 plus inches. After resizing the cables and struts to the initial NASTRAN load data the resultant deflection was 0.584 inches or less (74.2% less than the preliminary, see Appendix F). The penalty for the additional stiffness was a resultant final weight of 2.57lbs. (see Table 25).

CHAPTER 5

FINDINGS AND DISCUSSION

Description of Findings

Weight

Baseline and alternate design weights are shown in Table 26. The most light weight design was the optimized baseline design with a total weight savings of 56%. The weight saved could result in a revenue payload and is shown in Table 26 and approximated to cost $19,000 per pound saved (per flight). The lightest weight cable design was the 4-way grid coming in at approximately the same weight as the baseline design. The bi-directional and cable dome designs were approximately 37% and 48% heavier respectively than the 4-way grid.

The bi-directional grid data confirms Wang's assertion, see Chapter 2 (Properties of Contiguous Strut Tensegrity Grids), that "contiguous strut configuration with openings . . . are of low structural efficiency owing to the resulting isolation of struts, which results in cables sustaining tension in the compressive layer" [27]. In addition, internal loads in contiguous grids are greater than the traditional space truss, resulting in a tensegrity grid that is "40% heavier than that of the space truss" [27]. The 4-way grid, however, does not support this assertion, possibly due to its more efficient use of struts in bridging the outside of the 'opening'.

TABLE 26. Design Weight Summary

Design	Weight (lb)	Delta vs. Baseline	Price per Pound Savings ($19k/lb)
Baseline	2.436	0.0 (0%)	0
Baseline Optimized	1.082	1.354 (56%)	25,726
Cable Dome	3.794	-1.358 (-56%)	-25,000
Bi-Directional	3.520	-1.084 (-44%)	-20,596
4-Way	2.570	-0.134 (-0.5%)	-2,546

Cost

The costs for all designs are shown in Table 27. Engineering design hours were the clear driver, far outweighing material costs. It is clear that the more complex the design (and the more detail parts) the higher the cost. This is reflected in the fact that for the baseline design we only had only 12 detail parts required, while the cable dome, bi-directional and 4-way grids had 312, 577, and 679 parts respectively. Weight savings does offset the total hours (see price per pound savings data in Table 26), resulting in a lower

TABLE 27. Design Cost Summary

Design	No. of Parts	Cost (Dollars)				
		Material	Engineering	Total	Delta vs. Baseline	Delta Minus Weight Savings
Baseline	12	203	341500	341703	0 (0%)	0 (0%)
Baseline Optimized	12	203	341500	341703	0 (0%)	-25,726 (-8%)
Cable Dome	312	379	374750	375129	33426 (10%)	+58,426 (17%)
Bi-Directional	577	432	470500	470932	129229 (38%)	+149,825 (44%)
4-Way	679	225	623000	623225	281522 (82%)	+283,068 (83%)

overall cost for each alternate option for each flight flown (see delta minus weight savings in Table 27). The most economical option was the cable dome, primarily due to its reduced part count. The bi-directional and 4-way grids were, even with the weight savings offset, proved to be much more complex and costly.

Displacement

The shelf displacements (deflection) of the tensegrity grids are much greater than that of the conventional truss (baseline) design. Baseline maximum deflection occurred under –X liftoff loading condition. The alternate tensegrity designs experienced maximum shelf deflections under –Z liftoff loading. The tensegrity system with the least deflection was the Bi-Directional grid with a maximum deflection 193% greater than baseline. For the purposes of this design larger displacements take away from available shelf payload volume and have a negative impact on the design.

TABLE 28. Shelf Displacement Summary

Design	Load Condition	Maximum Shelf Displacement (inches)	Delta vs. Baseline (inches)
Baseline	-X Liftoff	0.179	0.0 (0%)
Baseline Optimized	-X Liftoff	0.179	0.0 (0%)
Cable Dome	-Z Liftoff	3.020	2.841 (1587%)
Bi-Directional	-Z Liftoff	0.525	0.346 (193%)
4-Way	-Z Liftoff	0.584	0.405 (226%)

Summary

Four different designs are examined and evaluated. The first conventional truss design was analyzed and optimized to achieve the lightest structure possible. The weight and approximate cost of the baseline was then compared to three alternate designs (cable dome, bi-directional and 4-way grid structure) that utilized tensegrity, or tensegrity like, systems.

The double layer tensegrity grids proved to have an even lighter weight, as expected, than the cable dome designs. The major drawback to these designs however was the complexity with almost 200% more parts than the cable dome design. In addition double layer grids have higher potential for stress concentrations and greater potential for an increase in detail part counts. If, however, this project were done on a larger scale, with more external support structure available, the weight savings cost may offset the complexity.

The cable dome structure proved to be the least costly of the alternate designs however it failed to provide weight savings. The cable dome design did, however, prove to be the most resilient with respect to point loading, effective load distribution and shelf displacement. The 4-way grid structure achieve the second largest weight savings, however its increased complexity make it undesirable from a cost perspective. If this project were done on a larger scale the weight savings, and therefore the cost offset, would clearly make the cable dome the winner. However, due to the limited weight saved the optimized baseline design, for this application, is the winner with reduced cost and an acceptable weight.

While tensegrity shows a weight savings potential (for the right design) its complexity, in the form of many parts to assembly and track, is its downfall. If this project was for a larger scale one of the tensegrity designs would be the clear victor. For tensegrity to complete for projects this scale the complexity must be reduced by reducing the part count and possibly assembling the structure as an integrated unit (utilizing additive metals or similar type process).

APPENDICES

APPENDIX A

PERSONAL CORRESPONDENCE

A.1. Correspondence with Ariel Hanaor.

Ariel Hanaor has performed extensive research in, and published on, tensegrity. He was a staff member at the Technion.

A.2. Correspondence with Mike Schlaich

Mike Schlaich is an architect at Schlaich Bergermann und Partner and structural designer of the Messeturm in Rostock (tensegrity tower).

A.3. Correspondence with Kenneth Snelson

Kenneth Snelson is an artist and one of the first patent holders for the tensegrity concept. He has dedicated much of his life's work to tensegrity sculpture.

A.4. Correspondence with David Campbell

David Campbell is an architect (Georgia Dome) and patent holder for cable dome type structures.

A.1. Correspondence with Ariel Hanaor

From: Ariel Hanaor [mailto:arielhanaor@gmail.com]

Sent: Wednesday, May 13, 2009 12:02 AM

To: Biele, Frank

Subject: Re: Permission to Publish

Dear Mr. Biele,

The answer to your question is a straightforward NO. It is always possible to design a conventional bar structure that is lighter than any tensegrity system. I am talking about spanning gravity loaded structures where, in tensegrity structures the chords are cables. This is an inherent property of the system and not just due to long compression bars. I attach my paper from the 5th International Conference on Space Structures, held in Uni of Surrey in 1993 (Thomas Telford, publisher), which deals with the topic of structural depth, a topic well worth developing, in my view. Incidentally, a cable dome is NOT a tensegrity structure. It is NOT a dome either (in the structural sense). It is a straightforward suspended cable structure, where struts simply serve as spacers between the supporting cables and the supported dome-shaped (but not dome acting) upper surface - see the above paper.

Lack of self criticism is a natural human frailty and particularly among engineers and scientists who tend to fall in love with their ideas. It takes courage to admit that a topic you have devoted a large part of your career to research has limited application. Tensegrity is a wonderful topic to research in view of the geometrical complexity and richness of configurations, but its practical application will always be limited to special cases such as space applications and applications of special visual effects (for which there is a price to pay). But the hell with practical application! Just have fun!

----- Original Message -----

From: “Biele, Frank” Frank.Biele@boeing.com

To: “Ariel Hanaor” <arielhanaor@gmail.com>

Sent: Wednesday, May 13, 2009 2:55 AM

Subject: RE: Permission to Publish

Dr. Hanaor,

Thank you for your timely reply and your interest in my thesis. It would seem the Thesis department (Library) and/or publishing house require additional letters of approval from the works author when reproducing any figures (The publishers, Rene Motro (editor International Journal of Space

Structures) and Bill Hughes (Director Multi-Science Publishing Co Ltd.), have already supplied a signed copyright approval)

I have been keeping abreast of the latest articles in tensegrity for years now and have read quite a few differing opinions, however the majority agrees that Tensegrity systems are not as efficient as the alternatives. I believe that we can agree that foldable tensegrity systems have shown their effectiveness in space applications (telescopes/reflectors for example), however, as you note, these structures have not been proven to be as efficient as their traditional counterparts as B.B. Wang notes,"Design results show that non-contiguous strut grid is much larger in internal forces, weight and deflection than contiguous strut grids, so are contiguous strut grids than the space truss except for the deflection aspect due to different material application." [1,P.62]

B.B Wang summarizes of your real-scale study of a flat tensegrity layout based on the triangulated simplexes ,"the self-weight of the geometrically rigid tensegrity grid is nearly twice that of the studied space grid." "...

long bars . . . is pointed out as the reason for the heavy weight of the tensegrity grids" [1,p.55] Alternatively, the bars could be made shorter, however, as Snelson notes, "short compression struts mean long tension lines which mean extreme elasticity. The struts can't be all that lightweight because they must support enormous compression loads. They need heavy and robust end-fixtures in order to absorb the powerful tension forces that pull outwardly with great cumulative force."[2]. And finally, a recommendation from Motro on the subject,"For sufficient rigidity, our experience in this field has shown that a rigidity ratio (EAstruts/EAcables) close to 10 is satisfactory. Above this, the behaviour (sip) is too flexible and leads to over sizing the cable elements. Below 10, the struts are overloaded and thus oversized." [3,P131]

It is important to note that the 'long bars' or struts, have been technologically advancing resulting in extremely light weight designs (utilizing composites) in areas such as space exploration (most notably with respect to end fitting and node design). I would pose to you the question as to whether, with advances in technology, Tensegrity systems would find a place in that architectural world as an efficient structure?

Tensegrity, it would seem, could be looked at as an attempt to manipulate the conventional rigid truss structure in a way that results in an efficient distribution of the load and, as a result, a reduction in the weight of the overall structure. While it is true that 'rigid' truss structure may be customized to give a similar resultant, the ability of the structure to deform (without yielding of the structure) is certainly limited wrt the tensegrity system (whether it be a triangulated contiguous (strut contacting strut) system or the more traditional tensegrity grids (k=1)). I understand the top cables of tensegrity systems slack structural depth is halved, and increasing prestress (preload) to compensate for this increases cable thickness and therefore system weight. I would ask whether there is a way to customize a tensegrity structure, as you would that of a conventional truss, so that the total structural weight is less than that of the truss? I would suspect that a tensegrity system that was lighter would exhibit large deflections that would not meet traditional requirements.

I have enclosed a pdf of the models that I am currently analyzing and reference R. Motro's book [3], and Kenneth Snelson's models for your reference in the last three pages of the attachment. As you can see the objective is to add stiffness to the entire system, the question is whether this is sufficient to hold practical loads in a launch environment (1-8g's) and will it compete with conventional designs.

Also, I am very much interested in reading your paper presented at the Space Structures Symposium at Surrey University. I will research your chapter in J.F. Gabriel's book

“Beyond the Cube”, and am also ordering “Geometrically Rigid Double-Layer Tensegrity grids”.

[1]Wang, B.B., Free Standing Tension Structures, Spon Press NY, NY, 2004.

[2].Jáuregui, Valentín Gómez, Estructuras Tensegríticas en Ciencia y Arte, Universidad de Cantabria, Santander, 2007, 200 pp. Also available in English: http://www.alumnos.unican.es/uc1279/Tensegrity_Structures.htm

[3]Motro, R., Tensegrity: Structural Systems for the Future, Kogan Page Science, Sterling, VA, 2003.

Thank you in advance for your time. I appreciate your interest and any feedback you can give me on the above.

Frank Biele

-----Original Message-----

From: Ariel Hanaor [mailto:arielhanaor@gmail.com]

Sent: Friday, May 08, 2009 9:26 AM

To: Biele, Frank

Subject: Re: Permission to Publish

Dear Mr. Biele,

Of course you may cite and use anything from any of my publications. I don’t think you need my permission but only that of the publisher.Regarding tensegrity structures as spanning structures (such as free-standing domes or planar grids) I am sorry to disappoint you but my work shows that these structures are inherently less efficient than conventional bar structures, due to the reduced effective structural depth. As top cables go slack structural depth is in effect halved. An additional flaw is the relatively long compressive struts. You can see some discussion of this in my chapter in J.F. Gabriel's book "Beyond the Cube" (John Wiley).A more specific discussion of the structural depth effect was presented in my last paper for the Space Structures Symposium at Surrey University - I don't have the details with me here at the moment (I am writing from home), but if you are interested I could email you a copy. I have not done any research on tensegrity structures since then, and in fact I am just about to retire both from my current position and from professional life as a whole.

All the best for your research

Ariel Hanaor

----- Original Message -----

From: "Biele, Frank" <Frank.Biele@boeing.com>

To: <arielh@techunix.technion.ac.il>

Sent: Friday, May 08, 2009 2:34 AM

Subject: Permission to Publish

Dr. A. Hanaor,

My name is Frank Biele and I am a graduate student in Aeronautical Engineering at California State University at Long Beach. I was writing to ask for permission to use the following in my Masters Thesis: Figure 6 (p.103) from your journal paper:

Hanaor, A., "Aspects of Design of Double-Layer Tensegrity Domes",International Journal of Space Structures Vol. 7, No. 2, pp101-113, 1992.

I have sent a copyright permission letter to International Journal of Space Structures c/o Multi-Science Publishing Co. Ltd. (see email below).

I have been intrigued by Tensegrity for over 9 years now, inspired initially by the Georgia Dome construction. My thesis is a comparison between conventional design (rigid bars and pinned struts) and tensegrity related designs (triangulated, Cabledome, and k=1 and k=2 ‘true’ DLTG’s (Double Layer Tensegrity Grids) using simplexes or contiguous struts (Contiguous struts = Tensegrity of the order of K=2, where the struts are permitted to be in contact with each other)).

I will also be referencing one of your other works in my Thesis:

Hanaor, A., “Prestressed Pin-jointed Structures-Flexibility Analysis and Prestress Design”, Computers and Structures,Vol. 28, No. 6, pp757-769, 1988.

Thank you in advance for your help.

Frank Biele

A.2. Correspondence with Mike Schlaich

From: m.schlaich@sbp.de [mailto:m.schlaich@sbp.de]

Sent: Wednesday, June 04, 2008 1:28 AM

To: Biele, Frank

Subject: Antwort: RE: Messeturm in Rostock

Dear Mr. Biele,

thank you for your response and your interest in the Rostock tower which I consider our Hommage to Snelson. I have asked our Stuttgart office to send you photos which do not carry copyright issues.

To me, adjustable cables on a structure like the rostock tower make no sense for several reasons. It would very difficult to adjust the turnbuckles as they only can be turned when there is no load on the cable. The cables are so short that the large turn buckles would make them look very heavy. Most importantly, it is practically impossible to adjust one cable without affecting the stress in all others, i.e. mistuning the entire structure. Finally, today it is possible to accurately calculate and fabricate cable-length so that later adjustment is not necessary.

A large field of application of “tensegrity” in a broader sense are “looped cable roofs” (spokes-wheels) roofs which Schlaich Bergermann und Partner have successfully used for many stadiums (see wwww.sbp.de). Towers and supports, I think, are generally too flexible to carry relevant loads.

I think that studying tensegrity is definitely worthwile as it exercises the mind and helps us learn to think in 3D. In this sense I admire what René Motro is doing in that field. There are countless theses and documents on tensegrity. A recent book published in Spanish is “Tensegridad” by Valentin Gómez Jâuregui, published at the Universidad de Cantabria in Spain. A Doctoral thesis on the subject is presently being terminated at the University of Weimar, Germany. Perhaps you would be interested in contacting the author, Mr. Wolkowicz (chrstian.wolkowicz@archit.uni-weimar.de)?

Best Regards, Mike Schlaich

Schlaich Bergermann und Partner

Beratende Ingenieure

im Bauwesen

“Biele, Frank” <Frank.Biele@boeing.com>

03.06.2008 19:34

An <m.schlaich@sbp.de>

Kopie Thema RE: Messeturm in Rostock

Dr. Schlaich,

I apologize for the delay (have been supporting Space Shuttle Discovery launch).I am completing my Masters Thesis in Aeronautical Engineering at California State University at Long Beach (CSULB) and wish to use some photograph’s of your

Messeturm in Rostock. As the messeturm can be considered art I would require permission to add a photograph of it to my thesis. I will be referencing you paper:

Schlaich, M., “The Messeturm in Rostock: A tensegrity tower”, Journal of the International Association for Shell and Spatial Structures, Vol. 45, No.2, pp 93-98, 2004.

I may also be referencing:

http://www.mero.de/uploads/tx_cwtcartoongallery/tens_tower_e.pdf

With regard to the assembly sequence I was wondering what the reason was for not using mechanically adjusting cables-was this a purely aesthetic decision?

Also, I wondered what you thought about the use of tensegrity structures and their future in architecture and design? From your Rostock paper (referenced above) you note that towers, “due to their inherent flexibility and irregularity of the geometry, it is doubtful that also in the future such structures will be much more than impressive sculptures”, and that tensegrities “only practical application has been the so-called “cable domes”.

Do you feel that College/Universities today offer undergraduate students an accurate picture of tensegrity and its possible applications, or do you feel that it is too specialized a field to be offered on the undergraduate curriculum?

Also, I was wondering what you thought of Rene’ Motro’s work on the subject? It is my intent to analyze contiguous strut tensegrity grids (k=2 and greater) and their usefulness in supporting larger structures.

Finally, if we were to recommend only two books (or papers) on tensegrity which two would it be?

My thesis is a comparison between conventional design (rigid bars and pinned struts) and tensegrity related designs (triangulated, Cabledome, and k=1 and k=2 ‘true’

DLTG's using simplexes. The support structure will be used in place of that currently being utilized to suspended avionics cold plate/shelf.

The two enclosed photos are what I would like to use for my thesis. (I am still waiting on permission from the photographer to use the enclosed photographs-have not received a response back)

I apologize for the lengthy request and questions. If you don't have time to answer the questions I understand and appreciate anything you can contribute.

Regards,

Frank Biele

A.3. Correspondence with Kenneth Snelson

From: Kenneth Snelson [mailto:k_snelson@mac.com]

Sent: Monday, October 13, 2008 6:10 AM

To: Biele, Frank

Subject: Re: Whitney Museum/ Tensegrity Thesis

Dear Frank Biel,

So that you have the correct perspective on my reason for patenting: my patents are solely for the purpose of publishing. I have never intended to get into litigation or proprietary legal matters in connection with my patents. In any case U.S. patents are valid for proprietary protection for only seventeen years. After that they are in public domain. My early patents have long been in public domain.

Since I'm not connected with a school or a society, the normal path to getting things published in journals, I have applied for several patents describing ideas that were novel at the time. Patents continue to be in publication as long the country exists and any citizen now can get a copy for free on the internet. Also, the patent examiners make a considerable effort to discover if the idea or principle is novel or merely something covered in someone's earlier patent.

You inquire why the "ZigZag tower" design doesn't appear in my Discontinuous Compression.... patent. The reason is that the patent did not aim at that kind of structure. It is about what its title says. Structures with what you are calling contiguous would not have pertained to the claims or disclosures in that patent.

Yes, in regard to the planar structures I sent pictures of, they are weave patterns. And of course they are "contiguous" structures, now that we are using that term. They are not very rigid; rather soft in fact. You might try to build one yourself and verify it by experience.

As for "YouSendIt", it's not necessary for the recipient to have an account. The person sending simply includes the recipient's email name. A notification is forwarded to you to download the files. And if you have a friend with a Mac he/she can open a Stuffit compressed file.

I look forward to seeing your thesis in November. It sounds very interesting.

Kenneth S.

On Oct 11, 2008, at 9:12 PM, Biele, Frank wrote:

Dear Kenneth,

Thank you for pointing out to me the fact that you built a Zig-Zag Tower in 1997 that appears to be exactly the same configuration as Mike Schalaich's Messeturm in Rostock (there was no credit given to you for the design in the IASS journal(Journal of the International association for Shell and Spatial Structures, Vol 45, issue 145, 2004)-although of course Mike does note that it is an 'homage' to you in my correspondence with him).

I see also that you emailed Burkhardt on a similar matter as well (http://bobwb.tripod.com/synergetics/photos/ken1.html). Upon further review of 1960-65 patent #3,169,611 figure 25 I have a question regarding the connection of the compressive elements in your Zig-Zag tower: While the patent develops and presents figures with "discontinuous compression, continuous tension characteristics" I was wondering where the integration of two compressive elements was mentioned (I just looked over the patent again and failed to find mention of this save the mention of, "A module . . . is an arrangement of compression members acting as the “bones” or skeleton . . . held in relatively rigid relationship to each other by a network of tension members . . . ” p1).

Regardless, clearly you developed the Zig-Zag tower in 1997 prior to the Schlaich Tower at Rostock(2003).

Regarding the photo’s I was hoping to use a picture of the X-piece model that you originally came up with and B. Fuller had conveniently ‘lost’ that was recently on display at The Whitney-see enclosed. Use of the X-module is also desirable. This email can only handle 4Mbyte attachments....I am not familiar with Stuffit or Yousendit (I just looked online and I could sign up for a trial account though. We also have a drop folder here at Boeing that you could drop the files into (I can send you information on that if you’d like- whichever is easier for you).

I am intrigued by your experimental planar structures from 1961, would the title be “woven planes” for both? They do pre-date the earliest pictures that I have found, most definitely, and they are indeed what some would refer to as k=2 tensegrity, or ‘contiguous’ tensegrity grids. I am curious to find out what experience you had with these: how rigid were they? Were they easy to assembly or difficult? What would your impression be for their use as a support system? Would you still classify theses structures as “tensegrity”?

Your concern over the use of your work is understood. My current thesis progress calls for the completion of my contiguous models within the next few days and then the analysis of the same models. I anticipate a completion of the preliminary write-up of the history and usage of tensegrity in mid-November and at that time will send you a copy to review and comment on.

I appreciate your continued interest in my thesis and look forward to your response.

Frank Biele

For your information I have included some pertinent research quotes on k=2 (k>1)/contiguous tensegrity grids:

“A Class k tensegrity structure for k > 1 allows k compressive members to be connected in a ball joint (so as not to apply torque from one member to another).” p1 [4]

From S. Jaun and J. Mirats [1]:

“-node on node: this method joints (sp) a node from one module with a node from another module. Such a structure does not comply with the definition of tensegrity proposed by Pugh. Even though, this new structure leads to the concept of contiguous strut tensegrity grid proposed later by Wang [ref]”P2 of [1]

From Wang [2]:

Isolation of struts in grid

“In non-contiguous strut tensegrity grids, struts are isolated among simplexes. The indirect force transfer leads to cables in tension in the compressive layer and infinitesimal mechanisms (or near-mechanism geometry) that enlarge the tensions, resulting in much-reduced resistant lever arm and low-stiffness.” . . . ”increases significantly the number of joints . . . ”p69

“ . . . contiguous strut tensegrity grids present much better structural efficiency over non-contiguous strut tensegrity grids.” . . . “It follows that structurally efficient grids should be at least based on contiguous strut configurations.”p69

Isolation of struts in simplex

“So if we expect that the resulting grids can be structurally efficient, struts should be allowed to be in contact in simplexes.”p.70

From Motro [3]

“Recently, Wang (1998) suggested using the expressions “non-contiguous” or “contiguous” tensegrity systems. This was interesting but not sufficient since these expressions pre-supposed that a chain of compressed struts can not be considered as a compressed component.”p26 Motro argues that his chain of compressed struts is one compressed element, however some definitions of tensegrity identify the end of the compressed element as the node, or locations where cables are attached.

References:

1. Juan, S., and Mirats, J., “Tensegrity frameworks: static analysis review”, Mech. Mach. Theory, 2007. doc:10.1016/j.mechmachtheory.2007.06.010

2. Wang, B.B., Free Standing Tension Structures, Spon Press NY, NY, 2004.

3. Motro, R., Tensegrity: Structural Systems for the Future, Kogan Page Science Sterling, VA, 2003.

4. Kanchanasaratool, N. and Williamson, D., Modeling and control of class NSP tensegrity structures, International Journal of Control, Vol. 75, No. 2, 123-139, 2002.

From: Kenneth Snelson [mailto:k_snelson@mac.com]

Sent: Friday, October 03, 2008 8:50 AM

To: Biele, Frank

Subject: Re: Whitney Museum/ Tensegrity Thesis

Dear Frank,

Yes you may use the photo of Needle Tower the others you need but I need to know how you intend to use them in your thesis. I've had too many disappointments in publications that turned out different from what I was told before they went into pring.

As you probably know Schlaich's tower is virtually a copy of my Zig-Zag Tower as shown on my website.

I've not seen the work of B. Wang. I found a site that shows what must be something of his that probably represent what you refer to as "contiguous" systems.

Here's a lo-res picture of the X-Module complex and the X-Piece. Which one are you referring to?

I'm also attaching two photos of experimental planar structures from 1961: woven planes. I think these pieces are much like Motro's "contiguous" stuctures in your pdf unless I misunderstand what's going on in those dim copies. I hope that you will include photos of these structures in your paper because they predate Motro's (or whoever did them) by a lot of years.

It's a grave nuisance that engineers more than once have characterized my work as "decoration", especially when they are copying me outright. It's either ignorance or an effort to dismiss what I'm about. Decoration is when you tie a ribbon around the neck of a poodle. Sculpture is a statement all by itself in three-dimensions. My statements are about the nature of structure, not too different from what engineers have attempted with tensegrity even though they talk about utility. Emmerich and Bucky had fantasies about buildings as have several others. Unworkable proposals never carried into actual buildings. It's for this reason I suppose that in journals my name is often omitted in an otherwise thorough bibliography because my "publications" are the sculptures themselves (plus the very descriptive and complete patent). I need to emphasize this fact because I've noticed it often over the years and as we know it's paper trail that survives. I really would like to know how you intend to handle these issues and what your thesis says about my work before I fax the permission form.

I'm sending these low resolution pictures just for identification. I have them in hi-res also and will send them. Can your server handle large files or should they go YouSendIt. I would compress them with Stuffit if you can open Stuffit packages.

Best,

Kenneth S.

P.S. Yes, all of the photographs are by me.

<Wood_X-Piece1948.jpg><Wood_X-Star1948-97.jpg><1960SnelsonBentTubeWeave.jpg><S60-PlanarPcRoofYorkAve.jpg>

On Sep 30, 2008, at 9:41 PM, Biele, Frank wrote:

Apologies on the first copy of the scanned in images, other is attached...but still fuzzy-hopefully it gets the point across with the figure in the lower half of figre 7.9 representing a typical single compressive element.

From: Biele, Frank

Sent: Tuesday, September 30, 2008 6:36 PM

To: 'Kenneth Snelson'

Subject: RE: Whitney Museum/ Tensegrity Thesis

Mr. Snelson,

Thank you for your timely reply. Apologies on the contiguous terminology, I am a structures design engineer and all the terms and acronyms are new and foreign to me as well. Contiguous struts = Tensegrities of the order of K=2, where the struts are permitted to be in contact with each other. I have enclosed diagrams from R. Motro's book [1] for your reference (this is what I intend to model for the my thesis). As you can see the objective is to add stiffness to the entire system, the question is whether this is sufficient to hold practical loads in a launch environment (1-8g's). Similar structure include Dr. Schlaich's Messeturm @ Rostock.

I was wondering what your thoughts were regarding these 'contiguous' tensegrity systems?

With regard to the photographs I thank you for the permission and was wondering if I could use the Needle Tower photo from your website (see photo enclosed from your sculpture section).

I was not able to find a picture of your X-piece on your site....would you be able to provide a photograph or link? If you can supply a photo can you also provide me with the name of the photographer (unless, of course, it was you!).

Enclosed is a standard permission form required by my University. If you would sign and either fax to 714-372-1484, or scan and send via email (I left the number for the X-Piece (I assume it's number 3, but wasn't sure what you had a photograph of) blank, as well as the date of it and would appreciate you either filling it in, or I can add that at and send it back to you if you prefer. Library services/ProQuest (their printer/publisher) will have your copyright permission on file.

My undergraduate advisor, and professor at Boston University, mandated simplicity and efficiency in design. One example was a technology applied to underwater vehicles allowing them to increase their speed exponentially by mimicking Sailfish; a clear illustration of the fact that we can learn from and integrate some of the systems or building blocks in nature (or say, Biology), much the same as Ingber's cell theory and your tensegrity sculpture (one naturally coming before the other, but that gets us into the whole chicken and the egg quandary). I believe that this is an important concept you have recognized in nature and as more science is applied doors will open for its application in structures design.

Thank you again for all your help and your time.

Frank Biele

REFERENCES:

1. Motro, R., Tensegrity: Structural Systems for the Future, Kogan Page Science Sterling, VA, 2003.

From: Kenneth Snelson [mailto:k_snelson@mac.com]

Sent: Monday, September 29, 2008 8:30 PM

To: Biele, Frank

Subject: Re: Whitney Museum/ Tensegrity Thesis

Dear Mr. Biele,

Thanks for your message about your thesis and all of its references. In a general way you are asking if statements I've made in the past are convictions I continue to hold. The answer is yes regarding the claims for engineering advantage of this kind of structure. As I've also said, since so many people have altered the definition of the word tensegrity for their own purposes, the word itself has little meaning. From the time Fuller declared --

absurdly in my view -- that all structures when properly examined are tensegrity, there's no way to agree on what it means. But I've said this over and over.

Re: contiguous systems, since I don't have the publications I don't know what is meant by the term.

About Emmerich, he visited with me in my studio in the 1970's. Later on we both wrote about our own histories regarding tensegrity or autotension in the "International Journal of Space Structures." We disagreed on the question of using these structures for big buildings.

Yes I agree with Schlaich's sketicism.

Yes, you may publish a picture of "Needle Tower" for your thesis and the X-Piece (3

Very best wishes for your thesis,

Kenneth Snelson

On Sep 29, 2008, at 12:54 AM, Biele, Frank wrote:

Mr. Snelson,

My name if Frank Biele and I am a graduate student at CSULB (California State University-Long Beach) that is completing (or 'trying to complete') my thesis on Tensegrity. My thesis is a comparison between conventional design (rigid bars and pinned struts) and tensegrity related designs (triangulated, Cabledome, and k=1 and k=2 'true' DLTG's (Double Layer Tensegrity Grids) using simplexes). The support structure will be used in place of that currently being utilized to suspended avionics cold plate/shelf (I have enclosed a brief overview of the Cabledome structure for your reference, modeling of the contiguous structure is currently underway). <<Model Views2A-wht .pdf>>

I had the good fortune of visiting NYC (I grew up on Long Island (Shoreham)) while your X-Piece (#3?) was on display at the Whitney Museum of American Art. I was pleased to see that they had, at a minimum, included your first tensegrity model and the now famous December 22, 1949 letter from Fuller (although I must admit it was difficult, at best, to try and decipher some of the words! REF your letter to R. Motro published in Nov 1990, International Journal of Space Structures [1]).

I have been intrigued by Tensegrity for over 9 years now, inspired initially by the Georgia Dome construction (cable domes, as I know now, "can not be considered tensegrity....they are, essentially, bicycle wheels." as you refer to them in your Aug 3, 2004 correspondence with Valentin Gomez Jauregui [2]). The credit for the invention of tensegrity could be compared to the somewhat more 'explosive' Physicist Lise Meitner's subjugation to Hahn Otto (1944 Nobel Prize for the discovery-who even after WWII refused to credit Meitner).

In R. Burkhardt's work "A Practical Guide to Tensegrity Design" [5] he touches on Ioganson and Emmerich:

"Some historians claim Latvian artist Karl Ioganson exhibited a tensegrity prism in Moscow in 1920-21 though this claim is controversial. Ioganson's work was destroyed in the mid-1920's by the Soviet regime, but photographs of the exhibition survive. French architect David Georges Emmerich cited a different structure by Ioganson as a precedent to his own work."[5, p.33)

In your letter to Maria Gough (dated June 17,2003)[2] you addressed Ioganson's IX model presented by Koleichuk in the 1992 Guggenheim show , "Koleichuk would have no way of guessing at the object, sticks positioned and strings properly attached, except that he had studied my work, or Bucky Fuller's or David Emmerich's."[2] I believe I know where you stand on Karl Ioganson, however in my research I do not believe that I have come across any comments from you on David Georges Emmerich who's French Patent includes the following description of 'Autoendantes':

“Self-stressing structure consist of bars and cables assembled in such a way that the bars remain isolated in a continuum of cables. All these elements must be spaced rigidly and at the same time interlocked by the pre-stressing resulting from the internal stressing of cables without the need for extra bearings and anchorage. The whole is maintained firmly like a self-supporting structure, whence the tern self-stressing.”[4]

My research led me to V.G. Jauregui’s “Tensegrity Structures and their application to Architecture”, and an overview of Rene Motro’s and B. Wang’s work [3] pushed me from my original plan of analysis of a cable dome to the use of contiguous strut grids. The following comments have influenced me:

From B.B Wang: “...contiguous strut tensegrity grids present much better structural efficiency over non-contiguous strut tensegrity grids.” ... “It follows that structurally efficient grids should be at least based on contiguous strut configurations.”[3,p69]

From Motro: “Recently, Wang (1998) suggested using the expressions “non-contiguous” or “contiguous” tensegrity systems. This was interesting but not sufficient since these expressions pre-supposed that a chain of compressed struts can not be considered as a compressed component.”[7,p26] Motro argues that his chain of compressed struts is one compressed element, however some definitions of tensegrity identify the end of the compressed element as the node, or locations where cables are attached.

From yourself, “short compression struts mean long tension lines which mean extreme elasticity. The struts can’t be all that lightweight because they must support enormous compression loads. They need heavy and robust end-fixtures in order to absorb the powerful tension forces that pull outwardly with great cumulative force.”[2]

I was wondering what your thoughts were regarding ‘contiguous’ tensegrity systems?

From your correspondence with Maria Gough (dated June 17,2003)[2] you note your thoughts on tensegrity:

“The unfortunate fact of tensegrity is not and never was functional except for the function in my sculptures or permitting viewers to admire the nature of pure structure. ... the forces in the system need to be so huge that the structure becomes inefficient for supporting any external loads.”

I was wondering if you still thought this way? It is undeniable that you have inspired those who have viewed your tensegrity systems (Donald Ingber included-see reference below), however with numerous advances in the state of the art (space elevators using ultra strong/thin composite thread) is this still a statement that you believe? Also, how does it feel to be associated with being the inspiration for the possible unlocking of the structural secrets of cells?

It may be of interest for you to know that I have been in contact with Dr. Mike Schlaich (Rostock Tower designer,Schlaich Bergermann und Partner (wwww.sbp.de)) and he thinks very highly of you, noting in one email of the Rostock Tower”...which I consider our Hommage (sp) to Snelson”[7]. Dr. Schlaich notes, and you may agree:

“due to their inherent flexibility and irregularity of the geometry, it is doubtful that also in the future such structures will be much more than impressive sculptures”, and that tensegrities “only practical application has been the so-called “cable domes”.[8]

“Towers and supports, I think, are generally too flexible to carry relevant loads.”[7]

My research led me to V.G. Jauregui’s “Tensegrity Structures and their application to Architecture”, and an overview of Rene Motro’s and B. Wang’s work [3]

In addition to the above references I also intend to include:

Donald Ingber, MD, PhD, professor and researcher at Children’s Hospital, and Harvard Medical School, credits Kenneth Snelson’s sculpture as inspiration for his life’s work in cell structure. In an Interview with Public Radio’s Studio 360 [6] Ingber recalls viewing Snelson’s “elegant” Needle Tower in 1975 as an undergraduate, and the way it reacted to stimuli (he knocked it). He was inspired to pursue integrity and later to identify its use in organizing cells through the cytoskeleton (Ingber, 2006). Merriam-Webster's Medical Dictionary defines the cytoskeleton (CSK) as the network of protein filaments and microtubules in the cytoplasm that controls cell shape, maintains intracellular organization, and is involved in cell movement.

I was wondering if you would allow me to publish a picture of your Needle Tower in my thesis (using as referenced above)? In addition I would very much like to also show your X-piece (#3) if possible.

I apologize for the length of the above and I appreciate any responses you can give to the above questions/requests, and realize that your time is valuable. I appreciate any assistance you can provide and Thank You in advance.

Frank Biele

List of above referenced works:

1."Correspondence with Kenneth Snelson" to R. Motro International Journal of Space Structures.November, 1990

2.Jáuregui, Valentín Gómez, Estructuras Tensegríticas en Ciencia y Arte, Universidad de Cantabria, Santander, 2007, 200 pp. Also available in English: http://www.alumnos.unican.es/uc1279/Tensegrity_Structures.htm

3.Wang, B.B., Free Standing Tension Structures, Spon Press NY, NY, 2004.

4.Emmerich, D., Contructions de Reseaux Autotendantes, Patent No. 1.377.290, 1963.

5. Burkhardt, R.,"A Practical Guide to Tensegrity Design" Version 2.27, [online], Cambridge, MA., http://bobwb.tripod.com/tenseg/book/, accessed Jan. - March, 2008.

6.Ingber, D., Lu Olkowski interviews Don Ingber, Studio 360 produced by Public Radio International and WNYC, Original airdate: May 12, 2006.

7. Personal Correspondence with Dr. Mike Schlaich (email), 6/4/2008.

8. Schlaich, M., “The Messeturm in Rostock: A tensegrity tower”, Journal of the International Association for Shell and Spatial Structures, Vol. 45, No.2, pp 93-98, 2004.

9. Motro, R., Tensegrity: Structural Systems for the Future, Kogan Page Science Sterling, VA, 2003.

A.4. Correspondence with David Campbell

From: David Campbell [mailto:dmc@geigerengineers.com]

Sent: Monday, May 04, 2009 5:10 PM

To: Biele, Frank

Subject: Permission to Publish in Thesis-David M. Campbell

Frank:

Please see the attached .pdf file- a signed permission form.

With respect to your questions, I have not had an opportunity to really consider this at time of writing. Please note the the behavior of these systems are quite dependent upon configuration and support conditions. I would be surprised if the radial non-triangulated Cabledome could be adapted reasonably to the configuration(s) you are working with. Triangulation of the network would no doubt be useful as would adoption of the double layer tensegrity grid.

I will try to give this more attention when I have more time to properly consider it.

Best Regards,

David M. Campbell P.E.

Geiger Engineers

2 Executive Blvd. Suite 410

Suffern, NY 10901

t 845. 368.3330 x 11

f 845. 368.3366

m 845. 729.1063

dmc@geigerengineers.com

From: Biele, Frank

Sent: Wednesday, April 29, 2009 4:42 PM

To: 'ddc@geigerengineers.com'

Subject: Permission to Publish in Thesis-David M. Campbell

David M. Campbell c/o Geiger Engineers,

Mr. Campbell,

My name is Frank Biele and I am a graduate student in Aeronautical Engineering at California State University at Long Beach. I was writing to ask for permission to use Figure 1 (Flowchart Illustrating General Approach to Tensile Membrane Structure Design and Engineering) from your paper "The Unique Role of Computing in the Design and Construction of Tensile Membrane Structures:", http://www.geigerengineers.com-, accessed April, 2009. You will find a permission form that is required to be filled out and signed.

<< File: David Campbell permission.doc >>

I have been intrigued by Tensegrity for over 9 years now, inspired initially by the Georgia Dome construction. My thesis is a comparison between conventional design (rigid bars and pinned struts) and tensegrity related designs (triangulated, Cabledome, and k=1 and k=2 'true' DLTG's (Double Layer Tensegrity Grids) using simplexes or contiguous struts (Contiguous struts = Tensegrities of the order of K=2, where the struts are permitted to be in contact with each other)). The support structure will be used in place of that currently being utilized to suspended avionics cold plate/shelf for use on the Space Shuttle.

I have enclosed a pdf of the models that I am currently analyzing and reference R. Motro's book [1], and Kenneth Snelson's models for your reference in the last three pages of the attachment. As you can see the objective is to add stiffness to the entire system, the question is whether this is sufficient to hold practical loads in a launch environment (1-8g's) and will it compete with conventional designs.

<< File: Prelim Model Views 4-29.pdf >>

I was wondering what your thoughts were regarding these 'contiguous' tensegrity systems, esp. with respect to traditional cable domes?

With respect to the paper you co-authored with Chen, Gossen and Hamilton:

Campbell, D., Chen, D, Gossen, P., and Hamilton, K., "Effects of Spatial Triangulation on the Behavior of "Tensegrity" Domes", Spatial, Lattice and Tension Structures, IASS-CSCE International Symposium 1994, published by ASCE, NY, NY, 1994.

I do understand that the conclusion of this paper was that radially oriented dome structures (cable domes) exhibited"...greater stiffness, much reduced to non-uniform and concentrated loads, an insensitivity to fabrication errors, as well as greater design flexibility of roof form than the triangulated dome system."[p662] Also noting that, "Generally, this added complexity [from triangulation] does not seem to yield any direct benefits other than a somewhat increased stiffness in response to load concentrations." It is for this reason that I have chosen to model a triangulated tensegrity structure that is contiguous (k=2).

Per the member loads shown in this paper it can be shown that overall the triangulated dome system results in members that see less stress (60%+) under the same loading condition as that of the cable dome. I hypothesize (until completing all the analysis) that, when analyzing the two systems as support systems (w/o a roof), the triangulated dome system utilizing a simplex (contiguous struts) will be most efficient (wrt loads and overall weight). While you assert the "triangulated system under uplift is attributable to the reversal in curvature in the ridgenet of the triangulated system." . . . and "The result is that the loss of tension in some cable elements is quite large." This is, instead, may be a load distribution issue which may be solved by using a simplex (double layer tensegrity grid). [P661]

I appreciate any response you can give to the above questions/requests, and realize that your time is valuable. I appreciate any assistance you can provide and thank you in advance for your help.

REFERENCES:

1. Motro, R., Tensegrity: Structural Systems for the Future, Kogan Page Science Sterling, VA, 2003.

Frank Biele

APPENDIX B

SPATIAL TRIANGULATION VS. RADIAL OREINTED DOMES

The paper entitled “Effects of Spatial Triangulation on the Behavior of “Tensegrity” Domes” compares circular, 394 ft. span, spatially triangulated and radial oriented dome structures, each with a dead load of 6.6 lb/ft^2. [23, p.653] This paper is reviewed below and analyzed so that the thesis model could be custom tailored to the Space Shuttle design condition. As a reference the approximate dead load of the proposed cable dome in this thesis is 8 lb/ft^2.

The study shows: Triangulation loads the hoop in uplift loads, compared to a (more effective?) distribution of the loads for a Cabledome. However, we see that for an unbalanced uplift on the triangulated dome the hoop tension can vary by “31% of the hoop tension, compared with a variation of 0.3% for the Cabledome” [23, p.656]

The authors of the paper note, “ . . . the triangulated structure is stiffer with respect to concentrated loads, at the expense of relatively large variation in element forces.” [23, p.661] Also,” . . . when both structures are subjected to uniform loads . . . Cabledome is significantly stiffer than the triangulated dome structure.” [23, p.661] For unbalanced loading the tables are turned.

For a uniform uplift load the max hoop stress seen in a triangulated dome is 4750. This number jumps from anywhere between 4114 and 6000 for an unbalanced uplift load. While both of these numbers are, on average, 36% and 28% respectively less than that of the Cabledome it does illustrate a weakness for dissipating unbalanced uplift. This fact makes the triangulated dome more desirable for uplift (or reversed) loading.

Hoop point loading (Load Condition 7 in the paper) in the –Z direction results in an 18% variation in tension for stay cables compared with 2.9% for the cabledome, however the deflection at this point is also 0.55 ft compared to 1.44 ft (almost 3 times as great) for a cabledome. This makes the triangulated dome more desirable for high stiffness applications.

The authors note, with respect to stiffness, “The triangulated system is not a (sp) stiff as the cabledome for uniform loads, but is somewhat stiffer in response to concentrated loads.” [23, p.660] For potentially critical point loading (which often is the result of a ‘one out’ load case (fail safe analysis)) the triangulated dome is clearly superior.

“The two structures behave differently in response to non-uniform loading, especially with respect to individual member forces. The Cabledome’s behavior is unique, member forces simply do not change much under the non-uniform load conditions evaluated.” . . . ”The nonlinear geometric stiffness contribution to the systems overall stiffness is quite large” [23, p.660]

It is possible that the authors incorrectly come to the conclusion that the “triangulated system under uplift is attributable to the reversal in curvature in the ridgenet of the triangulated system. The result is that the loss of tension in some cable elements is quite large.” This may be/is instead a load distribution issue. This is supported by the fact that under the same uplift the cabledome’s “center deflection is actually downward”. [23, p.661] One of the authors, David Campbell was asked about this issue and did not have time to respond specifically to the load distribution issue. He did, however note (in correspondence to the author, see Appendix A.3) that he had, “not had an opportunity to really consider this at time of writing. Please note the the behavior of these systems are quite dependent upon configuration and support conditions" (see Appendix A.4) A solution may be the addition of a simplex (double layer tensegrity grid).

The paper concludes that, "Generally, this added complexity [from triangulation] does not seem to yield any direct benefits other than a somewhat increased stiffness in response to load concentrations." . . . "The cabledome generally exhibits greater stiffness, much reduced to non-uniform and concentrated loads, an insensitivity to fabrication errors, as well as greater design flexibility of roof form than the triangulated dome system." [23, p.662]

Unfortunately for an application that is required to see potential point (concentrated) loading and, at the same time, is required to see reverse (-Z) loading with maximum stiffness (minimal deflection) the same conclusion cannot be drawn. It is for this reason that a triangulated tensegrity structure is utilized for the loading conditions in the Space Shuttle.

David Campbell concurs, "I would be surprised if the radial non-triangulated Cabledome could be adapted reasonably to the configuration(s) you are working with. Triangulation of the network would no doubt be useful as would adoption of the double layer tensegrity grid." (see Appendix A.4)

APPENDIX C

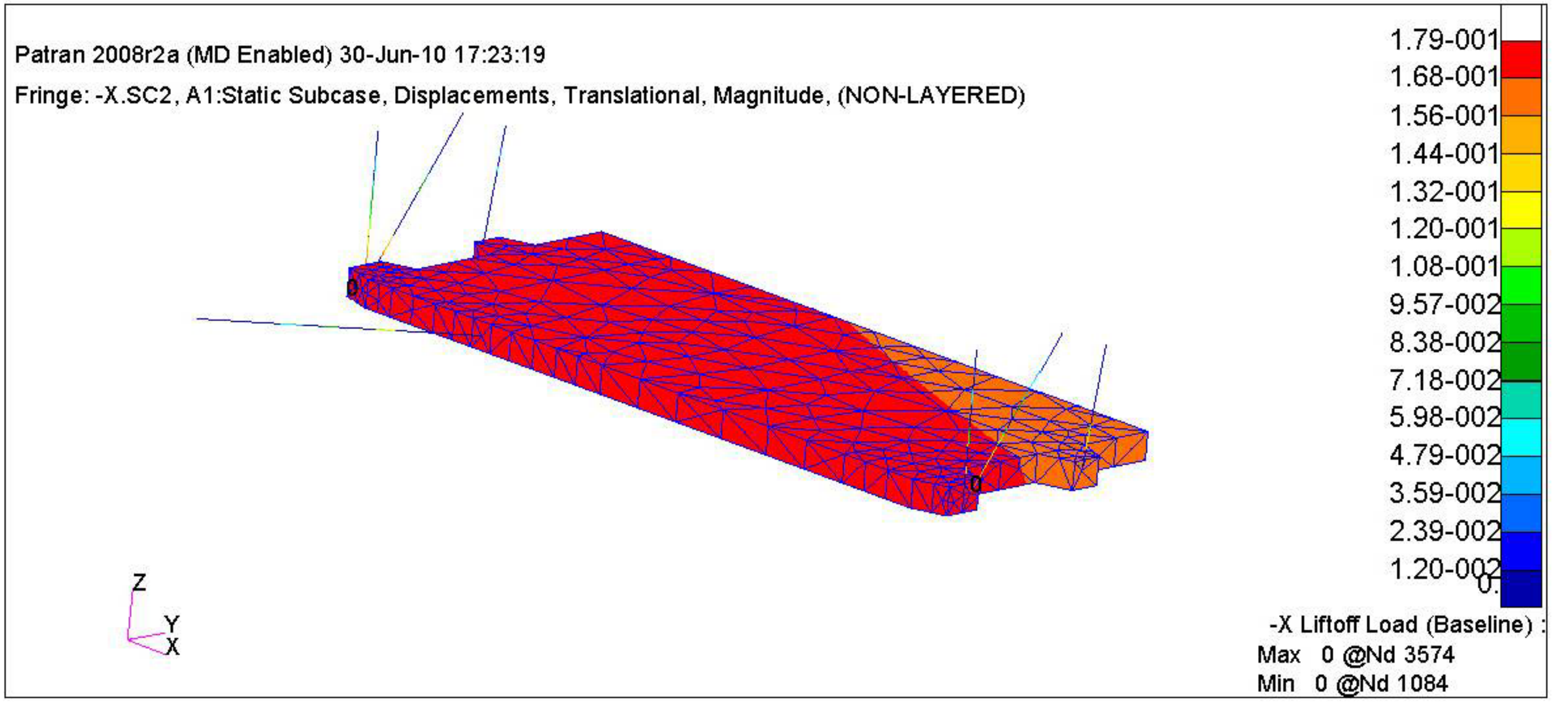
MARGIN OF SAFETY CALCULATION TABLE FOR BASELINE DESIGN

TABLE 29. Margin of Safety (M.S.) Calculations, Baseline Design (Optimized)

Strut Location	Element No.	Axial Compression Load (lbs)	Axial Tension Load (lbs)	F_{CR}	Z	Kc (Bruhn)	Local Buckling Stress	Rc (local buckling)	f_{CR} (Johnson-Euler)	Rc (Johnson-Euler)	Bending K-Factor	Fb ult	M_{ult}
1356 Inbd	2072	-1716	1896	270160	6466	1700	159468	0.0108	202620	0.1311	1.2728	160559	2539
1300 Inbd	2071	-1376	1520	263914	6619	1700	155781	0.0088	197935	0.1076	1.2728	160559	2539
Center	2070/4	-4194	2941	126945	13761	2900	127825	0.0328	95209	0.6821	1.2728	160559	2539
Outbd	2069/75	-2230	3181	94131	18558	3900	127468	0.0175	70598	0.4891	1.2728	160559	2539
Drag	2068	-5731	4018	88438	16599	3200	138087	0.0415	66328	0.7278	1.2727	160541	5812

Strut Location	Element No.	M_{END}	M_{MAX}	σ_b (Bending Stress)	Rb	Rb (Local Buckling)	Tensile Stress	Compression Stress	M.S.(non-local, Euler)	M.S. (local buckling)	M.S. (Tension)	M.S. (Compression)
1356 Inbd	2072	59.2	67.18	4247.93	0.0265	0.0266	33095	30813	5.35	25.74	3.02	3.45
1300 Inbd	2071	47.6	52.78	3337.48	0.0208	0.0214	26551	24643	6.79	32.05	4.01	4.56
Center	2070/4	161.1	372.34	23543.09	0.1466	0.1842	55718	88481	0.21	3.61	1.39	0.55
Outbd	2069/75	90.1	155.19	9812.70	0.0611	0.0770	54940	44342	0.82	9.58	1.42	2.09
Drag	2068	255.0	639.33	17659.91	0.1100	0.1279	40892	65932	0.19	4.90	2.25	1.08

FIGURE 38. Baseline maximum displacements (Biele, F.).



APPENDIX D

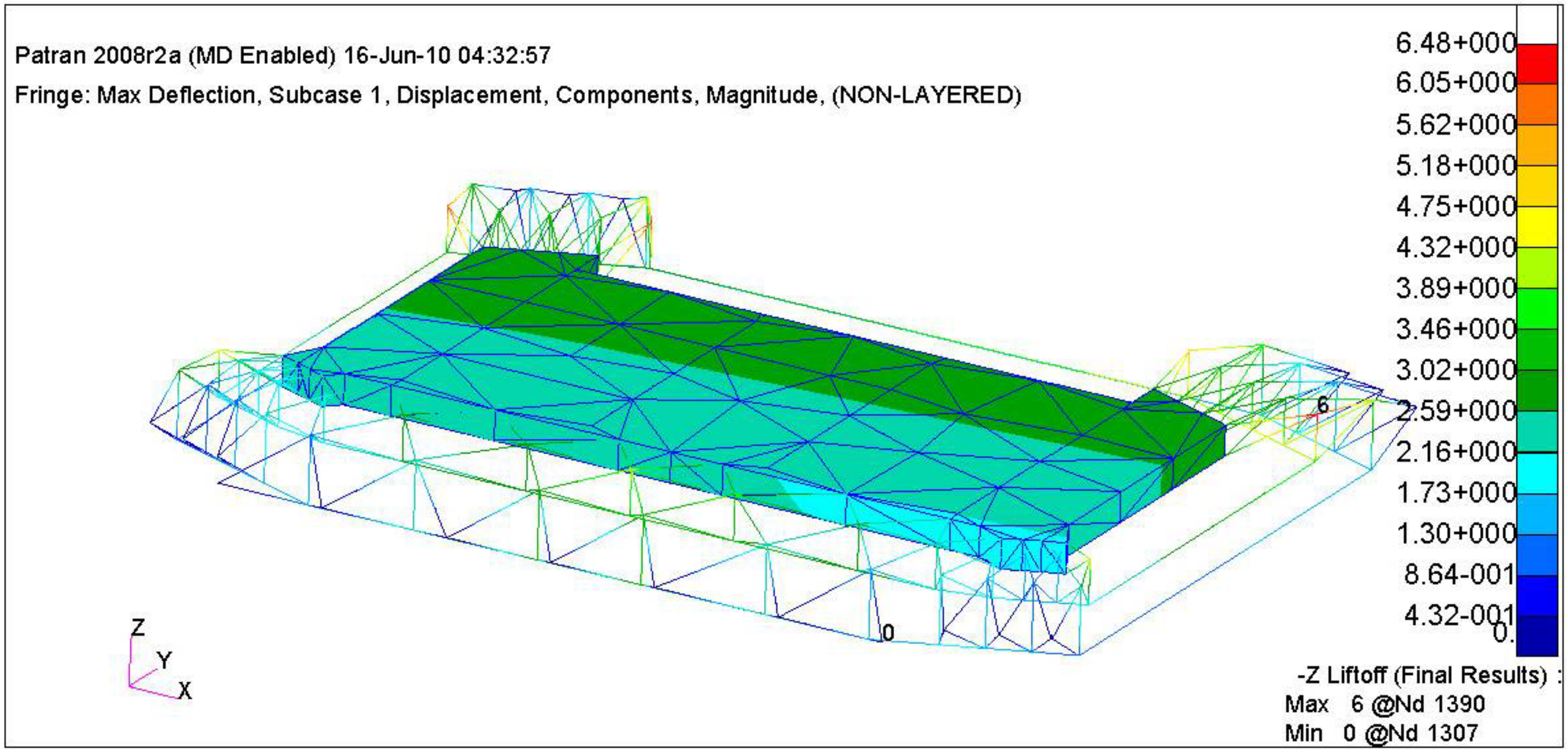
MARGIN OF SAFETY CALCULATION TABLE FOR CABLE DOME DESIGN

TABLE 30. Margin of Safety (M.S.) Calculations, Cable Dome

Description	Prelim Axial Compression Load (lbs)	Prelim Axial Tension Load (lbs)	F_{CR}	Z	Kc (Bruhn Table	Local Buckling Stress	Rc (local buckling)	f_{CR}	$f_{CR\ (johnson\text{-}euler)}$	Rc (Johnson-Euler)	Bending K-Factor	Fb ult	Mult
Outboard	-7876	0	528500	2001	10000	4880911	0.001614	396375	396375	0.3442	1.2722	160487	1680.57
Inboard	-8581	0	1181795	815	10000	13087839	0.000656	886346	886346	0.1541	1.2720	160463	1823.06
CABLES													
High Load		7623											
Primary Tension		3462											
Middle Tension		3727											
Inboard Tension		4488											
Hoop		7899											

Description	M_{END}	j	M_{MAX}	bending stress	Rb	Rb (Local Buckling)	MS(non-local, Euler)	Sectional Area (inches^2)	MS (local buckling)	tensile stress	M.S. (Tension)	Compression Stress	M.S. (compression)
Outboard	240.23	2.8192	344.1308	32863	0.2048	0.0067	0.8216	5.77E-02	118.8083	22941	4.7975	136428	0.0042
Inboard	248.84	2.8089	289.0857	25445	0.1586	0.0019	2.1983	6.28E-02	383.6468	21902	5.0724	136583	0.0031
CABLES													
High Load										7623	0.0100		
Primary Tension										3462	0.0100		
Middle Tension										3727	0.0100		
Inboard Tension										4488	0.0100		
Hoop										7899	0.0100		

FIGURE 39. Cable dome maximum displacements. (Biele, F.)



APPENDIX E

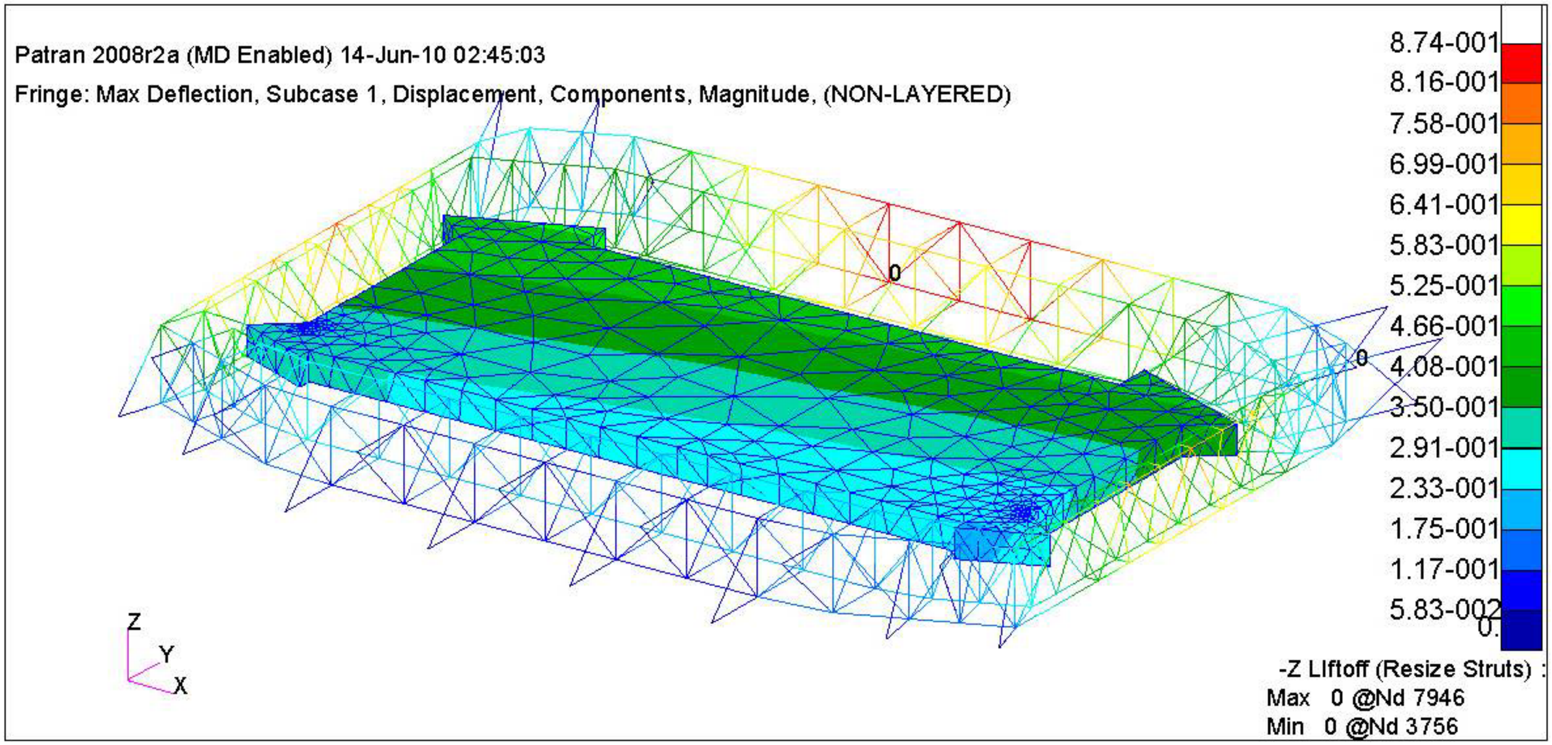
MARGIN OF SAFETY CALCULATION TABLE FOR BI-DIRECTIONAL DESIGN

TABLE 31. Margin of Safety (M.S.) Calculations, Bi-Directional Grid

Description	Prelim Axial Compression Load (lbs)	Prelim Axial Tension Load (lbs)	F_{CR}	Z	Kc (Bruhn Table	Local Buckling Stress	Rc (local buckling)	f_{CR}	$f_{CR\ (johnson\text{-}euler)}$	Rc (Johnson-Euler)	Bending K-Factor	Fb ult	Mult
Outboard													
Struts	-2224	0	275051	17200	10000	133645	0.01664	206288	206288	0.6624	1.2732	160597	525.07
Middle	-1327	0	344770	30312	10000	34629	0.038334	258578	258578	0.5107	1.2732	160602	377.99
Inboard	-1747	0	412237	16544	10000	96856	0.013706	309178	309178	0.4404	1.2732	160600	291.95
Hoop Struts	-865	0	398504	3488	10000	2163356	0.000808	298878	298878	0.4571	1.2726	160534	200.08
Primary Tension		3423											
Middle Tension		2636											
Inboard Tension		2525											

Description	M_{END}	j	M_{MAX}	bending stress	Rb	Rb (Local Buckling)	MS(non-local, Euler)	Sectional Area (inches^2)	MS (local buckling)	tensile stress	M.S. (Tension)	Compression Stress	M.S. (Compression)
Outboard													
Struts	73.19	3.1219	163.6805	50062.81	0.3117	0.3746	0.0265	1.63E-02	1.5560	22387	4.9409	136651	0.0026
Middle	44.07	3.7013	78.2032	33227.48	0.2069	0.9595	0.3935	1.01E-02	0.0021	18724	6.1033	132065	0.0374
Inboard	41.47	2.9028	66.9491	36827.73	0.2293	0.3802	0.4932	9.75E-03	1.5385	22813	4.8301	136160	0.0062
Hoop Struts	50.29	1.5141	82.9920	66589.28	0.4148	0.0308	0.1469	1.28E-02	30.6574	40354	2.2958	136627	0.0027
Primary Tension										308911	0.0100		
Middle Tension										308911	0.0100		
Inboard Tension										308911	0.0100		

FIGURE 40. Bi-directional gird maximum displacements. (Biele, F.)



APPENDIX F

MARGIN OF SAFETY CALCULATION TABLE FOR 4-WAY DESIGN

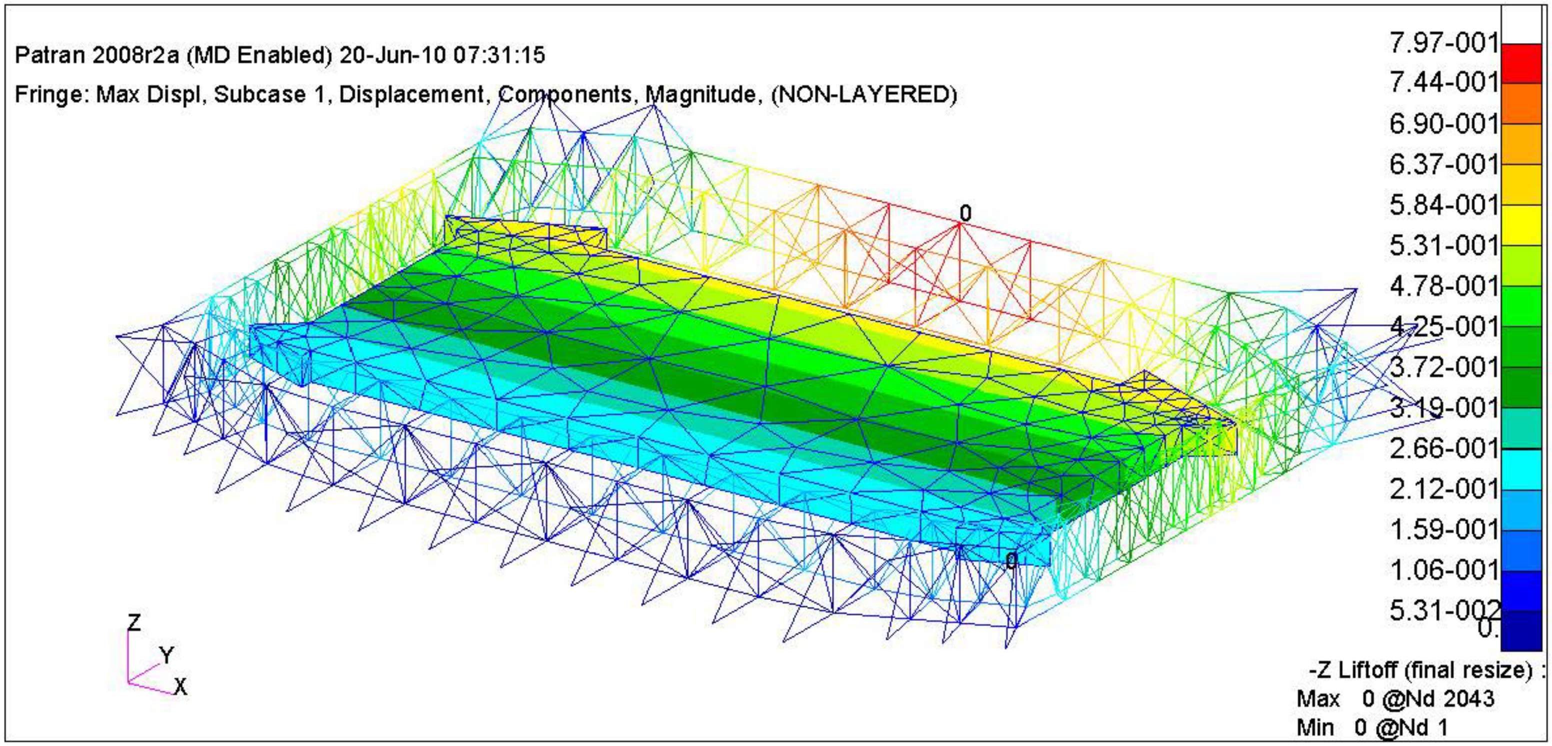
TABLE 32. Margin of Safety (M.S.) Calculations, 4-Way Grid

Description	Prelim Axial Compression Load (lbs)	Prelim Axial Tension Load (lbs)	F_{CR}	Z	Kc (Bruhn Table	Local Buckling Stress	Rc (local buckling)	f_{CR}	$f_{CR\ (johnson-euler)}$	Rc (Johnson-Euler)	Bending K-Factor	Fb ult	Mult
Struts													
Outboard	-1200	0	398268	30444	10000	29748	0.04035	298701	298701	0.4349	1.2732	160602	358.71
Middle	-646	0	185248	40585	10000	35850	0.018033	138936	138936	0.6207	1.2732	160600	206.64
Inboard	-1179	0	295094	19357	10000	98623	0.011957	221320	221320	0.6114	1.2732	160599	225.95
Hoop	-588	0	371671	10437	10000	267715	0.002196	278753	278753	0.5336	1.2732	160594	65.67
Outboard Diagonal	-873	0	305692	32214	10000	34565	0.025262	229269	229269	0.6831	1.2732	160601	237.91
Middle Diagonal	-785	0	388351	29763	10000	31909	0.024595	291264	291264	0.3142	1.2732	160602	203.15
Inboard Diagonal	-790	0	303966	25816	10000	54020	0.014624	227974	227974	0.6495	1.2732	160601	147.26
Outboard Cables		1288											
Middle Cables		918											
Inboard Cables		923											
New High Load Cable		1627											

TABLE 33. Margin of Safety (M.S.) Calculations, 4-Way Grid

Description	M_{END}	j	M_{MAX}	bending stress	Rb	Rb (Local Buckling)	MS(non-local, Euler)	Sectional Area (inches^2)	MS (local buckling)	tensile stress	M.S. (Tension)	Compression Stress	M.S. (Compression)
Struts													
Outboard	39.51	3.8528	63.3274	28352.8	0.1765	0.9531	0.6354	9.24E-03	0.0066	17688	6.5193	129914	0.0545
Middle	21.46	3.3575	44.8421	34850.62	0.2170	0.9721	0.1937	7.50E-03	0.0099	16679	6.9739	86237	0.5886
Inboard	36.98	2.5210	76.1472	54124.1	0.3370	0.5488	0.0544	8.72E-03	0.7833	26287	4.0595	135307	0.0125
Hoop	16.93	1.4694	29.0059	70934.32	0.4417	0.2650	0.0253	4.35E-03	2.7431	41392	2.2132	135298	0.0126
Outboard Diagonal	28.31	3.3188	49.8972	33683.46	0.2097	0.9745	0.1200	7.53E-03	0.0002	19111	5.9594	115968	0.1814
Middle Diagonal	24.88	3.2358	39.0904	30903.08	0.1924	0.9685	0.9740	6.43E-03	0.0070	19668	5.7621	122135	0.1217
Inboard Diagonal	24.56	2.4405	48.7523	53168.44	0.3311	0.9842	0.0198	5.90E-03	0.0011	26785	3.9654	133969	0.0226
Outboard Cables										308911	0.0100		
Middle Cables										308911	0.0100		
Inboard Cables										308857	0.0102		
New High Load Cable										308854	0.0102		

FIGURE 41. 4-Way gird maximum displacements. (Biele, F.)



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